station, and furnished to the Weather Bureau through the courtesy of Prof. E. W. Hilgard, director, Berkeley, Cal. No corrections have been applied to the records. Pomona, Cal., is in latitude N. 34° 3'.

Hourly percentages of possible sunshine near Pomona, Cal.

Month.	Percentages of possible sunshine recorded during the (local mean time) hours ending—														Hours of sunshine.		
	A. M.							P. M.									
	6	7	8	9	10	11	Noon.	1	2	3	4	5	6	7	Total actual observed.	Total possible.	Percentage of possible.
1896.															*H'rs.*	*H'rs.*	
January ..	....	....	20	49	56	66	66	56	69	70	71	36	....	....	176.0	316.2	56
February .	....	4	52	73	86	90	89	87	88	88	88	74	11	....	238.6	318.5	75
March.....	....	16	53	65	73	76	77	72	68	70	67	62	22	....	222.9	372.3	60
April... ..	6	51	67	69	80	77	84	81	76	80	84	85	69	7	273.3	391.6	70
May.......	3	46	68	78	82	90	85	81	84	85	83	85	79	10	297.1	432.6	69
June	7	33	43	62	74	81	87	94	96	96	99	96	93	30	296.7	431.5	76

KITE EXPERIMENTS AT THE WEATHER BUREAU.

By C. F. MARVIN, Professor of Meteorology, U. S. Weather Bureau.

[Continued from the June REVIEW.]

EFFICIENCY.

Hitherto no exact and scientific methods appear to have been employed to determine the relative merits of different kites, or to fully measure and analyze their action. Experimenters in general have been contented to make a rough estimate by eye of the angular elevation attained, or if this has been measured the results, with rare exceptions, have been inaccurate, and the observations limited to a very small number. Often, probably, but a single reading has been made at a favorable moment when the kite had momentarily attained an extreme elevation. Moreover, the observations have generally been made with the object of ascertaining the altitude of the kite when a long length of deeply sagging line was out. Little or no notice appears to have been given to the effect of the long line in modifying the angular elevation of the kite. If any accurate measurements of the behavior of kites have been systematically made such measurements have, with one or two exceptions, been conspicuously absent from any published accounts of kite experiments known to the writer. It is therefore impossible to form any estimate of the relative merits of the kites employed by different individuals. Eye observations without the aid of instruments suffice to determine only general qualities of steadiness, etc. Those factors upon which the usefulness of a kite for meteorological purposes depends, namely, the *lift* and *drift*, can be determined accurately only by aid of instrumental measurements. Eye estimates of the angular elevation of kites tend nearly always to exaggerate the amount of the angle, and data of this sort respecting the behavior of kites can have no place in scientific investigations.

Various methods of expressing numerically the merit of a given kite may be employed. The *lift* and *drift* may be made the measure of excellence of a given kite. But the *lift* and *drift* of a kite vary with every gust of wind, and it is difficult to deduce from these quantities a true numerical rating of the merit of a kite under examination. This objection to the use of lift and drift as a measure of excellence would have less weight if the wind blew with a steady direction and constant force, but this is never the case. Moreover the *lift* and *drift*, aside from depending directly upon the force of the wind, depend further upon both the actual surface of the kite and upon the angle of incidence. A very perfect kite which happened to be bridled in such a fashion that the angle of incidence was, for example, 25°, would, in all probability, show a smaller lift and a larger drift than a much inferior kite bridled so that its incidence was 15°. This difference of incidence would, in all probability, wholly escape the notice of an ordinary observer unless his attention was specifically directed to discover it. Even if discerned with the eye the real numerical relation could be established only by carefully made instrumental observations. The *lift* and *drift* in themselves, therefore, do not constitute a suitable basis for a true numerical estimate of the useful effect available in a kite. They are in fact only conventional and derived ideas. We must go back of them to the fundamental forces from which they are derived for the basis upon which true comparisons can be made. *Efficiency* is the technical term widely employed in all branches of engineering to designate numerically the useful effect available in machines of any sort. Thus, we have the efficiency of a steam engine, of a boiler or furnace, the efficiency of electric generators, motors, converters, etc., so likewise we may have the efficiency of kites. This measure of merit, as adopted at the Weather Bureau for the comparison of kites with each other, is based upon fundamental mechanical principles, and is widely applicable to any kind of kite. The resulting measure is not directly dependent upon the angle of incidence of the kite or upon the direction or force of the wind.

Efficiency of kites.—The basis upon which any rating of efficiency is deduced is very largely a matter of choice. In dealing with machines and appliances for producing physical or mechanical effects, economical considerations have much to do with the ultimate or absolute utility of the devices employed. From the economic standpoint an efficiency rating is an exceedingly complex result, depending upon many factors of the most heterogenous character — cost of space, wages of employees, cost of transportation, interest on investment, etc. These factors can be related to each other only in a highly arbitrary and empirical manner. The efficiency of mechanical devices, as the term is ordinarily used, is not generally deduced upon the economical basis but depends upon purely mechanical and physical considerations of cause and effect. Dismissing economics we will likewise define the efficiency of kites upon the physical and mechanical basis. Even here, choice may be made among several methods. We may consider that the most efficient kite is one which can attain the highest elevation. As we shall see hereafter, the elevation attained by a kite is purely a question of the forces acting upon the string. It is very plain that to make the efficiency of a kite depend in any way upon the string is not desirable. Even if we eliminate, as we may, effects due wholly to the string, and make the efficiency of the kite depend upon its power to attain elevation, we still make a bad choice, for we would thereby fail to consider that kites may be employed for other purposes than attaining elevations. A highly efficient kite from such a standpoint would be highly inefficient if it were employed to pull sleds or carry a line ashore from a stranded vessel.

A basis upon which the efficiency of a kite can be deduced, that is not open to such objections as raised above, may be had by considering only the *inclination of the total resultant wind pressure* to the surface of the kite. A kite, fundamentally, is a surface either plane or curved against which it is designed the wind shall press. The ideal kite is that surface; the actual kite is a material substance having thickness, edges, possibly a tail, etc. The string is an entirely separate accessory not necessarily included in discussing efficiency. In the analysis of the action of the wind upon surfaces a principle of efficient action was pointed out on page 162,[1] as follows: "*The condition of ideal efficiency (that is, an efficiency of 100 per cent), in the action of the wind upon thin plane surfaces, obtains when the total resultant pressure is exactly normal to the surface.*" Recognizing that a kite is a surface against which the wind shall press, we say broadly that the pressure is most efficiently exerted when for plane surfaces the total pressure is exactly normal to the surface. For arched sur-

[1] MONTHLY WEATHER REVIEW, May, 1896.

faces we must deal with inclinations to a tangent, or more conveniently to the chord of the arch. We will speak of this more in detail further on.

The reader who has followed the section on the "Analysis of forces" in the May REVIEW (page 157) and who has in mind the effects of the weight of the kite as set forth on page 203 of the June REVIEW is prepared to readily understand the application of the above-mentioned principles to the derivation of the efficiency of a kite. Under ideal conditions, that is, conditions in which edge pressures, surface or skin friction, waviness and fluttering, eddy effects, etc., are wholly absent, it follows as a direct consequence of the principles already established that the ideal kite, whose weight is considered inappreciable as compared with the wind pressure, will fly in such a manner that the direction of the string next the kite will make an angle of 90° with the surface of the kite or with the longitudinal axis thereof. In the case of an actual kite of appreciable weight and more or less imperfect in other respects, it will be found upon measurement that the direction of the string next the kite will make an angle of less than 90° with the longitudinal axis. This angle between the direction of the string next the kite and the longitudinal axis of the kite is properly made the numerator of the efficiency ratio, and for convenience and brevity we will call it hereafter the *efficiency angle*. It is the angle $A\ O\ R$ in Fig. 65. If, upon measuring the angle between the direction of the wire and the kite, it were found to be 75°, for example, then the efficiency of the kite would be given by the ratio of this angle to 90°, that is—

$$\text{Efficiency} = 75 \div 90 = 83\tfrac{1}{3} \text{ per cent.}$$

This measurement relates specifically to the *position* the kite takes in the air, and does not deal with the *pull* of the kite. We might, therefore, more specifically call the above defined efficiency the *position efficiency*. The *pull* is a factor wholly independent of the *position* when we consider simply the mechanics of a kite, and it is well to keep these factors separate in estimating the merits of kites.

The different positions that kites of different efficiencies assume when flying from a string which is either so light or so short that it does not sag to an appreciable extent is shown in Fig. 65. $A\ B$ represents the midrib or longitudinal axis of a kite; and the string is supposed to make an angle of 75° therewith, corresponding to a position efficiency of $83\frac{1}{3}$ per cent. The angle of incidence of the horizontal wind with the kite is supposed to be 20°. In such a case the angular elevation of the kite will be 55°. If, however, the kite were perfect, in which case the efficiency angle would be 90°, the position the kite would then take is shown at $A'\ B'$, and its angular elevation would be 70° instead of 55°, the kite still retaining the same angle of incidence of 20°. It might be argued that by changing the angle of incidence of the kite $A\ B$ by the proper amount without changing its efficiency it would fly as high as $A'\ B'$. This may be true, but the more efficient kite would pull harder, and if *its* angle of incidence were likewise changed, the perfect kite would again fly higher than the imperfect kite and pull equally hard.

The foregoing treatment of the question of the position efficiency of kites applies strictly only to plane surface kites, and throughout all preceding discussions where efficiency angles have been measured in reference to a midrib or longitudinal axis of the kite it has been assumed, as was generally the case in the Weather Bureau kites, that the *apparent angle of incidence* was also the *true angle of incidence*.[1] If this is not at least approximately so in a given kite, or if, as in a trapezoidal kite, the sustaining surfaces are at different angles of incidence, then the efficiency angles must be taken in reference to the planes themselves.

Arched surfaces.—When we deal with arched surfaces some experimental results show that the wind forces in question do not act in the same manner as upon plane surfaces, and while the general principles involved in deducing efficiency still remain the same, a slight change in computing it numerically will probably be required, owing to the fact that in the ideal case the string might form with the longitudinal axis an angle—$A\ O\ R$, Fig. 65—greater than 90°.

The difficulty in the case of arched surfaces is that we do not know, a priori, the maximum possible angle between the string next the kite and the surfaces, or the chord of the arc; that is, we have no certain value for the denominator of the efficiency fraction. Some observations show that the angle ought to be greater than 90° in the ideal case, but just how much greater is not known. This is a matter which is at present of minor importance. In fact, this angle undoubtedly varies with every modification of the curvature of the arch, and possibly with changes in the angle of incidence. While, therefore, we may not be able to arrive at a mathematically correct numerical value of the *efficiency ratio* in the case of arched surfaces, we still have in the *efficiency angle* alone a wholly satisfactory basis for numerically rating the merit of any kite, whether with flat or with arched surfaces. The most efficient kite, other things remaining the same, is the one showing the maximum efficiency angle. The experiments up to July 1 had not been carried sufficiently far to show the most satisfactory procedure in the case of arched surfaces. The foregoing remarks refer to the *position efficiency* of kites. Let us consider briefly the pulling power of kites.

Pull.—In comparing the pulls of different kites, the comparison must, of course, be made always for the same conditions; that is, for the same velocity of the wind, the same angle of incidence, and the same unit of surface. There is very little reason why kites should differ much in the pull per square foot of surface if we have been careful to measure the sustaining surface upon a systematic basis, such as already explained in the REVIEW for June, page 201. The following appear to be the principal causes why one kite should pull more than another under otherwise similar conditions: Arching the surfaces of the kites, as we have already explained, may increase the pull very greatly. In kites of the cellular type the sheltering of one surface by another may diminish the pull per unit area, more or less. The pervious character of ordinary cloth may serve to diminish the pull. The wind may not press to good advantage upon the pointed lateral and bottom extremities of such kites as the Malay, and the pull may be less in consequence.

Efficiency—how determined.—Having defined the mechanical significance of the efficiency of kites, the next point is how shall the necessary measures be made in order to compute the efficiency in a given case. The only quantity which it is necessary to measure is the angle between the wire and the kite. It would not be difficult to construct a small recording instrument which, when connected between the bridle and the main wire, would produce a continuous record, from which the angle between the main wire and one of the bridle lines could be deduced. Since the angles between the bridle and the kite may always be known, the record mentioned would suffice completely to give the desired efficiency angle. This sort of an instrument could be combined with a small dynamometer recording the pull of the kite upon the same record sheet with the efficiency angle. If still further combined with a recording anemometer, the resulting apparatus would constitute a complete *kite indicator*, since it would give the principal elements required in working out the efficiency of kites and the action of the forces thereon. It was not considered advisable to attempt to introduce such an instrument for recording the elements mentioned, although the matter received serious consideration, and the dynamograph portion of the instrument for recording the pull of the line, either at the kite or at the

[1] Page 201, MONTHLY WEATHER REVIEW, June, 1896.

reel, was actually constructed. This instrument is shown in Fig. 68 and is described on page 241.

Incidence scale.—In the absence of the instruments required for making the above described automatic record of the efficiency angle, another method was devised for measuring by eye observations, not only this angle, but the angle of incidence of the kite and, simultaneously, its angular elevation. This method is best explained in connection with a kite with rectangular cells. By aid of a stencil made from a sheet of oil-board paper a series of graduation lines 1 inch apart are boldly marked in black upon the white cloth of one of the upper sustaining surfaces of the cell, usually the forward cell, as shown in Fig. 66. The lines are one-quarter inch broad, and each fifth line is about 2 inches longer at each end than the intermediate lines, which are about 4 inches long.

The zero line of the scale is at the front edge of the cell. Figures need not be applied to any of the lines, as the grouping in fives renders the reading of the scale sufficiently easy and certain. The scale, for convenience, may be called the incidence scale, since by its use we ascertain the angle of incidence of the kite.

When a kite of the usual proportions provided with such a scale is flying in a normal manner, and is viewed from a position near the reel, a part only of the incidence scale is visible, the remainder being concealed behind the lower surface of the cell. At a distance of a few hundred feet the number of divisions of the scale exposed to view can be read with the unassisted eye, but in our regular experiments a small reading telescope, such as employed by physicists for reading galvanometer scales, etc., has been used. The telescope for the purpose was mounted upon an ordinary engineer's tripod. Easy motion in both altitude and azimuth was provided, and in the absence of a regular vertical circle an accurately divided draughtsman's protractor was arranged to give the angular elevation of the axis of the telescope. Assisted by the telescope, readings of the incidence scales have been made with as much as 2,000 feet of wire out, but in order to eliminate from the observations as much as possible the effect of the sag in the wire, which had to be taken into account in the manner hereafter described, observations were nearly always made at distances of between 400 and 1,000 feet.

The protractor was divided to half degrees, and readings of less than this amount could be made. Owing, however, to the constant and great changes of the position of the kite, refinement in angular readings, when working at short range, possess no significance. For the same reasons the estimates of the incidence scale were confined in general to half inches. To offset the coarseness of these measures observations were repeated at intervals of from 30 to 60 seconds, and ten or more readings made in each set from the mean of which the final deductions were made.

The act of making an observation consists in bringing the kite in view in the telescope, and following its motions until at a favorable moment a reading of the scale can be satisfactorily made with the kite near the center of the field. The inclination of the telescope at this moment is the angular elevation of the kite, which is thus determined simultaneously with the scale reading. Fig. 67 shows the relation of the angles in question. The angle A at the kite is the observed angle of elevation; i is the desired angle of incidence; the angle x is given by the equation:

$$\tan. x = \frac{s}{h}$$

in which h is obtained from the known height of the cell and s is the reading of the incidence scale.

Finally, $i = 90° - (A + x)$.

If we were justified in neglecting the sag in the wire, then the efficiency angle between the wire and the kite would be—

Efficiency angle = elevation + incidence.

Generally, however, we will desire to be more accurate than to neglect the sag in the wire. The data for making the necessary allowance for the sag of the wire is obtained if, at the moment the scale reading is made with the telescope, an assistant observes the inclination of the wire at the reel. In a subsequent section the mathematical equations of the curve assumed by the kite wire will be discussed at length, and it will be shown that when the sag in the wire at the reel is known the sag next the kite can be found. For the present we will call these angles S' and S, and they are so marked in Fig. 67. With the kite at a distance of 400 feet or more from the reel, lines of sight, such as $R\ V$ and $R\ V'$, will be sensibly parallel, although they are not so in the drawing, owing to the exaggerated size of the kite. In practice, observations are made only when the sag in the wire is slight, in which case the angles S and S' are nearly equal to each other. Owing to the peculiar character of the curve assumed by the wire, the angle S will be smaller than S' as a rule. The efficiency angle, including the sag, is

$$A + i + S.$$

Inclination of wire at reel.—As stated above, the sag of the wire is obtained from a measurement of the inclination of the wire at the reel. This was measured by means of a protractor, arranged to hang over the wire with its diameter parallel thereto, and provided with a light hand or index pivoted at the center of the arc and always assuming a vertical direction, thus serving to indicate on the graduated arc the angle of inclination of the wire. This angle subtracted from the angular elevation of the kite, measured from a point carefully chosen just at one side of the reel, gives the angle S'. In strong winds the position of the index of the protractor was sometimes affected, and it was necessary to weight the index with a small plumb-bob. Finally, the whole protractor was inclosed in a glass case.

Probable errors.—By means of the telescope and incidence scale simultaneous observations of the angular elevation and incidence of the kite are made in a highly satisfactory manner. Owing to the great variations of the wind the incidence is found to vary considerably, as also the position of the kite. Observations must be made quickly and at favorable moments. The measurement of the incidence angle is less accurate in proportion as the scale reading is small. An error amounting to a whole inch in a single reading of the scale can not be made except by gross mistake, and the error of the mean of several readings is probably less than 0.5 of an inch. The corresponding error in the angle, under conditions found in practice may, in extreme cases, be as much as 2°. Repeated observations of the same kite on different days have been so consistent with each other that it is believed the errors are actually less than those just described. If a satisfactory measure is not obtained in the manner described it is necessary simply to move the telescope back from the reel a short distance, so as to obtain such an angle of view as $T\ T'$, Fig. 67, resulting in more accurate measures. If efficiency tests are to be made at the same time, then an additional measurement of the angular elevation of the kite from a point near to and at one side of the reel will also be required.

General remarks on efficiency.—The manner we have chosen for deducing the efficiency of a kite is such that the weight of the kite is a modifying factor, causing the efficiency to be less than would be the case if the efficiency were made to depend only upon such imperfections as edge pressures, skin friction, waviness, eddies, etc. To include the effect of the weight with that of the imperfections just mentioned is, we believe, a very proper course, inasmuch as the kite must first sustain its own weight before it is available for rendering useful services. Moreover, if for analytical purposes it is desired to study separately the imperfections mentioned above, the precise knowledge we may always have of the weight of a kite enables us, by the aid of simple mechanical principles and the resolution of forces, to perfectly separate the effects due to weight and other disturbing influences, so that each may then be studied separately.

Weight and efficiency.—On page 203 of the June REVIEW the modifications produced in the direction of the string next the kite, due to the weight of the kite and different wind velocities, were fully pointed out. We now notice also that every change in the angle of the string means a corresponding change in the efficiency angle, which is the angle $A\ O\ H$, $A\ O\ H'$, $A\ O\ H''$, etc., Fig. 63.[1] From a consideration of these points we see that owing to effects arising from its own weight the efficiency of a kite in light winds is less than in heavy winds. In Fig. 63 it was assumed that the direction of the resultant pressures $O\ Q$, $O\ Q'$, $O\ Q''$, etc., corresponding to increasing wind forces, remained always at the same angle with the kite surface. This will be the case when the influences due to edge pressures, waviness, eddies, etc., follow exactly the same law of increase as obtains for the normal wind pressure. This seems likely to be the case with edge pressures, perhaps, but it is probable that the detrimental effects of eddies and fluttering are proportionally greater at high than at low velocities. It may, therefore, happen that a kite seriously defective in respect to these last-mentioned imperfections would, with moderate wind forces, show increasing efficiency up to a certain point, but that in still stronger winds the efficiency would actually become less. In other words, *the strong wind would seem to blow the kite down.* Such an instance has not come within my own observation, but its probability is easily seen from a physical standpoint.

Incidence and efficiency.—The pressure of the wind upon the kite may be feeble, not alone because of light wind velocities, but also by reason of the kite flying at small angles of incidence. If the incidence is made too small the pressure of the wind even at considerable velocities will be only a relatively small multiple of the weight, and this condition, as we have found, results in only small angular elevations. There is, in fact, a particular incidence giving a maximum effect. This is treated of further on, in the section on the catenary.

Ascending air currents.—Thus far it is assumed, in computing the incidence and efficiency of kites, that the wind flows in horizontal streams. This is generally, but not always, the case. It is well known that masses of air generally have a descending or ascending as well as a horizontal motion. Under these circumstances the actual direction of motion of the air may be in lines that are upwardly inclined to an appreciable extent. Kites are very sensitive to such conditions and the action of such ascending currents causes the kite to soar up to an unusually high angular elevation. The keen observer will not be misled into believing, as some have, that the phenomenal behavior of a kite under such influences is due to some peculiar excellence of the kite itself. These effects of ascending currents were well known and understood by the scientific kite flyers of half a century ago. A brief quotation in regard thereto is cited in the April REVIEW, page 114, mentioning the experiences of the Franklin Kite Club.

If a kite flying normally in a horizontal wind assumes an angle of incidence of, say 15°, then in an ascending current flowing in a direction inclined upwardly at an angle of 10° the same kite would seem to assume an angle of incidence of only 5° and would soar to a point near the zenith, although still flying at an angle of incidence of 15°.

When the bridle adjustment of a kite remains fixed, the angle of incidence of the kite will also remain constant with a given wind force. Even with different wind forces, unless they are very feeble, the incidence will change, but very little. Furthermore, the efficiency angle of a given kite is a definite angle, which must remain nearly constant in the same kite so long as it is not modified in any way or the wind force is not too feeble. Since, as we have just seen, the incidence and efficiency angles of a kite must be constant with given conditions, it necessarily results that the angular elevation will also be constant. When, therefore, we have fully established the constants of a given kite by careful measurements under normal conditions of longitudinal air motion, the behavior of the kite under abnormal conditions of ascending currents is, perhaps, one of the best measures we have of the amount of the abnormality. By means of a kite with its constants carefully determined, it thus seems possible to measure, with a fair approximation, the upward inclination of movements of masses of air otherwise quite inaccessible.

Causes of small efficiency.—We have found that when the wind pressure is several times the weight of the kite the influence of the weight on the efficiency angle is very small and unimportant. Results obtained with good kites under favorable conditions show that efficiencies of 90 per cent and over may be attained. When, therefore, we find, under favorable conditions of wind, smaller efficiencies than this, we know at once that the kite is either excessively heavy or defective in respect to edge pressures, waviness, eddies, etc., or the angle of incidence is too small, which latter is easily corrected by changing the bridle adjustment. An incidence of 15° is probably as small as can be employed with advantage, at least with flat surface kites. In the case of cellular kites, if the top and bottom surfaces are too near each other, or if the front and rear cells are too close together, the flow of the air through the structure of the kite may be, as it were, choked up to a greater or less extent. All such effects will have a direct influence on the efficiency.

From these brief remarks it is evident that in dealing with efficiency we have a powerful and searching artifice for numerically and justly expressing the merit of a given kite. It is hoped experimenters will familiarize themselves with the principles involved and apply them in general to kites of their own, so that some idea can be had of the real duty that a given kite has performed.

GENERAL OBSERVATIONS OF KITES.

While the measurements of the angles referred to in the preceding section are sufficient to establish the angle of incidence at which a given kite is flying, and to determine its position efficiency, still other observations are needed to ascertain all the facts we wish to know concerning the behavior of the kite. Among these the following are discussed:

Measurement of the tension of the wire.—Prior to July all measurements of tension of the wire at the reel consisted of eye readings of a spring scale attached to the reel in the manner described in the April REVIEW, page 122. The scale of the dynamometer employed embraced 50 pounds, and when the tension on the wire was greater than this limit a purchase (in the mechanical sense) was obtained by use of a movable pulley, the dynamometer being attached to one end of the cord passing over the pulley. This tackle, as is well known, multiplies effects by two; hence, the dynamometer which indicates normally only 50 pounds answers for a maximum strain of 100 pounds.

Dynamograph.—Fig. 58 represents a small dynamograph devised to give an automatic record of the tension of the wire. The clock is one of the very small, inexpensive house clocks on sale by any jeweler. But very little alteration is required to mount the clock on its hour-hand axis, which, being suitably prolonged, is clamped firmly in the bearings $A\ A$, with the result that the whole cylinder containing the clock revolves at the rate of one revolution per hour. In order to

[1] Fig. 63 will be found in the WEATHER REVIEW for June.

reduce to a minimum the motion of the moving parts concerned in measuring the tension, the spring employed is exceedingly stiff, being one of the excellent springs commonly used in steam engine indicators. A strain of 100 pounds compresses the spring about one-sixth of an inch. This motion is magnified and recorded with precision by the pen in a manner readily understood from the figure. The dynamograph in its original form was designed for use with small kites with pulls of not to exceed 35 pounds, whereas experiments were actually made requiring a greater range of scale. The necessary modifications in the dynamograph to adapt it to larger scales were not, however, made until after July 1.

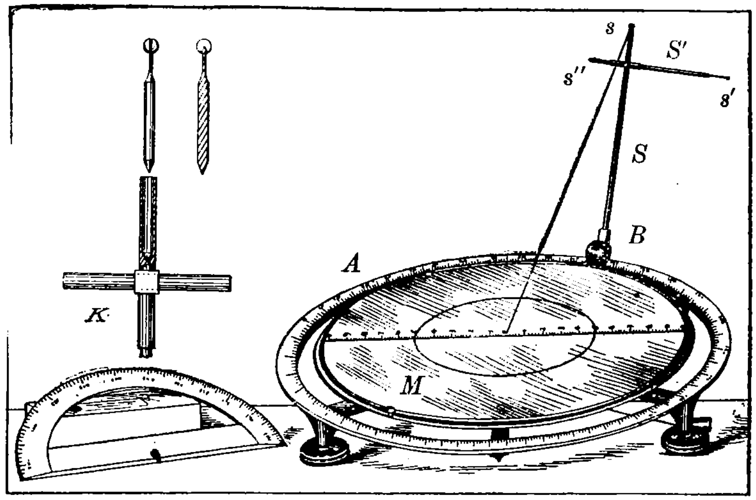
Measurements of wind velocity.—No direct measurement of the wind velocity was made during the kite experiments except the continuous records made at the Weather Bureau. These records answered every purpose so far as the general experiments were concerned, but a much more specific and local measurement is greatly needed in order to formulate the laws connecting the pressures per unit area with the angles of incidence, velocity of wind, perviousness of cloth, character of kite, etc. A small anemometer weighing only 0.8 of a pound has been constructed which records, not by the usual step by step methods, but continuously every movement of the cups. Fractions of a mile at their true momentary velocities are fully recorded by it and momentary velocities for very brief periods have been deduced with the same accuracy as is attained in ordinary velocity measurements. This instrument was, however, not available for use until after July and its further description is reserved to accompany the publication of results we hope to attain by its use.

Measurements of angular elevation.—The manner of measuring the angular elevations of kites by aid of the telescope, as described in the section on efficiency, is not the most convenient when nothing but the angular elevation is needed nor is its accuracy all that can be desired in the case of lofty ascensions. Two other methods have therefore been employed.

Nephoscope.—It was often desired to ascertain the average position of a kite without observing necessarily its efficiency. Owing to the constant changes going on in the angular elevation of the kite the average must be based on numerous measurements made momentarily and at perfectly equal intervals of time. The best results are secured if the instrument employed admits of being read or at least set at a precise instant of time. This is the case with the nephoscope employed by the Weather Bureau for observing the positions and motions of clouds. It is shown in the illustration below and was described at length in the Weather Review for January, p. 9.

Its manipulation is so simple that scarcely more than one second is required for ascertaining the angular elevation of a kite. The nephoscope is mounted upon a firm table or support near the reel and the mirror *M* carefully leveled by the aid of an ordinary level which accompanies the instrument. To observe the kite the eye is placed so that the former is seen reflected from the central spot of the mirror, and the sighting knob *s* on the staff *S* set so that the knob is also seen reflected at the center of the mirror. This setting can be made in a very short time. The angle of the inclined thread may then be measured with the protractor, and we have the angular elevation of the kite. Such settings of the nephoscope were generally made at exact intervals of thirty seconds for a period of five to ten minutes. The average of ten or twenty readings of this sort may be considered to give a close measure of the average position the kite under examination will take under ordinary conditions of atmospheric motion. Experimenters should not be satisfied with a less exact and truthful record of the average performance of a given kite than one obtained in some such way as that described.

Sextant.—The nephoscope answers admirably for the measurement of angular elevation under most circumstances. In the case of lofty ascensions, however, the kite appears very tiny and is sometimes difficult to see. In order to measure the angular elevation accurately under such circumstances a sextant fitted with a low-power glass has been employed. A small plate-glass mirror about 12 inches square, mounted on three leveling screws, was used in place of the ordinary artificial horizon of mercury. The accuracy of this method of measuring the angular elevation is really more than demanded. It was not necessary to read the vernier of the graduated scale at all, as sufficient accuracy was attained by eye estimation of the minutes of the scale. By the optical principles involved in the use of the sextant with an artificial horizon the actual scale-reading gives double the angular elevation. At great heights the apparent position of a kite varies but little, nevertheless our practice has been to read angles at comparatively short intervals, so that a fair average position may be attained.



Marvin's Improved Nephoscope.

Calculation of height.—When the sag in the wire is disregarded the altitude of the kite is given by the equation:

$$H = r \sin. A.$$

When r is the length of wire out and A is the angular elevation of the kite. This assumes that the length of a straight line from the reel to the kite is the same as the length of the wire itself, which of course can not be true. If, however, the sag in the wire is not over 20° at the reel, then roughly the straight line will be only about 2 per cent shorter than the wire. For a sag of 30° the difference will be about 4 per cent. The height, computed by the equation given above, should then be diminished by the proper percentage allowance for sag. Results obtained in this way will be quite as accurate as by more complicated methods of deducing the height by triangulation or by records of air pressure obtained from barographs attached to the kites. Other accurate methods of computing the height will be given in a subsequent section on the properties of the catenary, including the case of invisible kites.

RESOLUTION OF FORCES.

When the efficiency angle and pull are known for a given kite, also the bridle adjustment, we have the data for constructing a complete diagram of the actual forces acting on the kite. By way of illustrating more in detail how the analytical observations on kites have been conducted in the Weather Bureau investigations, and to show how the diagram of forces is constructed and the resolution of forces carried out in an actual case, the following observations from our field notebook are given:

Tests of kite No. 30, *May* 19, 1896 (*see Table VI, Review for June, p.* 201).

[Bridle adjusted as shown in Fig. 69; observations made with 700 feet of wire out.]

Time. p.m.	Pull.	Incidence scale.	Inclination of wire.	Elevation of kite.
h. m. s.	*Pounds.*		°	°
2 4 15*	20	2.5	....	60.5
2 8 20	20	3.5	56.0	57.5
2 9 20	25	3.5	57.0	60.0
2 10 45	4	6.0	50.0	61.0
2 11 30	20	3.2	54.0	57.0
2 13 00	20	3.0	55.0	58.0
2 13 50	22	4.5	56.0	57.5
2 14 30	24	4.0	57.0	60.5
2 16 20	16	3.8	56.3	59.0
2 18 20	15	4.0	56.0	59.5
2 20 00	14	4.0	55.0	56.5
2 22 20	10	6.0	51.0	55.0
....	24	5.0	54.0	55.0
Means....	17.8	4.21	54.8	58.0

*This first observation, being incomplete, is omitted in taking the sums and means.

Results.—The height of the cell of this kite is 20 inches; therefore, the scale reading, 4.21, corresponds to an angle x given by the equation

$$\tan. x = \frac{4.21}{20} = 0.2105 \text{ whence } x = 11.9°.$$

Hence, the incidence $= 90° - (58.0° + 11.9°) = 20.1°$.
Sag of wire at reel $= 58.0° - 54.8° = 3.2° = S'$.

When the sag of the wire is small, as in this case, a close approximation to the sag at the kite, that is, the angle S is given by taking $S = 81\%$ of S', therefore $S = 2.6°$.

Hence, the efficiency angle $= 58.0° + 20.1° + 2.6° = 80.7°$.

$$\text{Whence the efficiency} = \frac{80.7}{90} = 90\%.$$

This kite was observed later on the same day both by the telescope and nephoscope with 2,000 feet of wire out. The readings are given below.

Time. p.m.	Pull.	Incidence scale.	Inclination of wire.	Elevation of kite.
h. m. s.	*Pounds.*		°	°
3 44 10*	12	7	35	50.5
	23	6	45	52.0
	23	5	46	55.5
	18	5	46	58.0
	26	4	50	54.0
	30	4	50	54.0
	8	5	48	55.5
	24	5	45	52.5
	18	4	44	52.5
3 49 00*	18	5	47	53.0
Means....	20.0	5.0	45.6	53.75

* Time of only first and last observations noted.

From these observations $x = 14.0°$; incidence $= 22.2°$; $S' =$ sag at reel $= 8.2°$; $S = 6.6°$; efficiency angle $= 82.6°$; efficiency $= 92\%$.

From observations with the nephoscope the following results were obtained—2,000 feet of wire out:

Time. P. M.	Pull.	Inclination of wire.	Elevation of kite.
h. m s.	*Pounds.*	°	°
3 54 00*	16	42.0	52.0
	14	42.5	51.5
	10	46.0	53.5
	8	42.0	55.0
	16	41.0	51.5
	16	44.0	51.5
	8	40.0	52.5
	16	41.0	49.5
	10	47.0	55.0
	8	39.0	54.0
	12.2	42.45	52.60

* Every 30 seconds.

If we assume, as we are justified in doing, that the average incidence of the kite was the same as actually observed in the observations made a few moments before we shall have, incidence assumed to be 22.2°, sag in wire at reel, $S' = 10.2°$; $S =$ approximately 8.1°; efficiency angle $= 82.9°$; whence the efficiency $= 92\%$.

We have given above three separate sets of observations. The amount of variation in the efficiency angle that may be looked for under such conditions is shown in the three values, 80.7°, 82.6°, and 82.9°.

The pull on the wire was measured at the reel where it is less than the tension at the kite. The difference between the two will depend upon the relative inclination of the wire at kite and reel. The mathematical relation between the tensions at different points on the kite wire does not concern us at the present moment and is reserved for treatment in a subsequent section.

Diagram of forces.—Fig. 70 shows the actual diagram of forces corresponding to the results obtained from the first set of observations. The center of gravity of the kite is at the center of figure as at g. Passing a line through F so as to intersect the axis of the kite at the efficiency angle, viz, 80.7°, we have the line $L\ F\ O\ R$ which is the action line of the resultant of all the forces at the kite. To resolve this total resultant force into its components we draw a vertical line, $g\ O$, through the center of gravity of the kite and lay off thereon from O downward the line $O\ G$, representing on a convenient scale the weight of the kite=3.59 pounds. From properties of the catenary it can be shown that when the tension of the wire at the reel is 17.8 pounds as observed in the present case the tension at the kite under the observed conditions will be 21.1 pounds. This force is represented by the line $O\ R$ drawn to the same scale as $O\ G$. Completing the parallelogram of which $O\ R$ and $O\ G$ are the diagonal and one side, respectively, we have the line $O\ Q$ which represents the total resultant of all the wind pressures upon the kite. By measurement we find this resultant to be 24.2 pounds and by prolonging its action line downward we find that it intersects the kite at an angle of 85.1°.

We wish now, from this diagram, to arrive at some idea as to the relative intensity of the wind pressure upon the front and rear cells of the kite. The front cell is freely exposed to the wind, while the rear cell is in some degree sheltered, and we may reasonably expect to find the pressures on the latter deficient. When we wish to represent by a single force the combined effect of the wind pressures upon both the upper and lower surfaces of a cell, the principles of mechanics lead us to locate the point of action of that single force midway between the surfaces, provided the upper and lower pressures are equal. If they are unequal, then the point of action must be proportionately nearer the greater force.

Now, in such a kite as that under consideration the upper and lower surfaces are separated by a distance a little greater than their width. In such a case it is believed the upper surface at ordinary wind velocities can not be sheltered to any large extent by the lower surface, and that the pressures on the two surfaces are sensibly equal, at least in so far as concerns the interference of one surface with the other. Nevertheless, in the case of the rear cell it is quite probable that the exposure of at least the upper surface is far from unobstructed, and the pressure of the wind upon the lower surface also may be slightly deficient by reason of the proximity of the front cell. Therefore, it is probable that the points of action of single forces representing the combined pressures upon the upper and lower surfaces of the cell can not with accuracy be placed midway between the surfaces; but our present purposes do not require that these points be located with great accuracy. It can be shown that little or no sensible error will be produced in the results we seek if we assume, as we shall, that the points of action of the single forces in question fall midway between the upper and lower surfaces of each cell, as, for example, upon the line $C\ C$ in Fig. 70.

Returning now to the resolution of the forces, we found from the diagram that the line $O\ Q$ represented the total wind pressure upon the kite. This force is made up of the pressures upon the individual cells, and we have just found that the points of action of the pressures upon the individual cells may be assumed to fall upon the line $C\ C$. In accordance with the principles of mechanics, the point of action of the total resultant pressures will also fall upon the same line, $C\ C$. The point, in fact, will be at the intersection of the lines $O\ Q$ prolonged, and $C\ C$; that is, at O', and the total resultant is completely represented by the line $O'\ Q'$. This force, as just stated, is the resultant of two forces, one being the wind pressure upon the forward cell, the other the corresponding pressure upon the near cell. Since the sustaining surfaces in the cells are equal, the wind pressures ought also to be equal, under the assumption that one cell does not shelter the other. Our diagram of forces enables us to discover the difference in the pressures on the two cells. To prevent confusion of lines, we will use in this study the diagram in Fig. 71, which represents the line $C\ C$ of Fig. 70, and the force $O'\ Q'$. Before we can divide the force $O'\ Q'$ into the two parts representing the wind pressures upon the front and rear cells, respectively, we must locate the centers of pressure in those cells. We can not do this very accurately, but the points of action of the forces are undoubtedly forward of the middle of the cell in each case. Several formulæ based on experimental work have been given for computing the position of the center of pressure on a rectangular plane surface, and if we employ one of these we can not go very far astray. Chanute in "Progress in Flying Machines," gives the formula $d = l\ (0.2 + 0.3 \sin. i)$. Applied to the present case, l is the width of the cloth bands in the kite under consideration, and i is the angle of incidence; d is the distance from the front edge of the surface to the center of pressure. Computing the result, we find $d = 5.8$ inches. By this method we locate the center of pressure in each cell at P and P'. Through the points thus found we draw the lines $P\ N$ and $P'\ N'$ parallel to $O'\ Q'$ and proportional to the lines $O'\ P'$ and $O'\ P$ respectively. According to the well known principle of the lever, two forces represented by the lines $P\ N$ and $P'\ N'$ will be exactly equivalent to the single force $O'\ Q'$, and *vice versa*. That is to say, if $O'\ Q'$ is known, then the forces $P\ N$ and $P'\ N'$ are those sought, and represent the forces on the two cells respectively. $P\ N$ is a pressure of 18.5 pounds, while $P'\ N'$ is only 5.67 pounds, which shows to how great an extent the rear cell is sheltered by the forward cell. If we assume that the action of the wind upon the forward cell is unimpeded and acts sensibly with the maximum effect, then the rear cell experiences only 31 per cent as much pressure as the front cell. In other words, the efficiency of the rear cell is only 31 per cent. These results depend in a manner upon an assumed position of the center of pressure within the cells. But any other logical assumption that one may desire to make concerning the position of the center of pressure will lead to results that do not differ greatly from those found above, and a noticeable disparity between the pressures upon the front and rear cell will still exist. If the center of pressure is placed nearer the center of each cell than we have assumed, then the disparity will be greater. If it is placed at the extreme front edge of the cell, which would be absurd, there would still be some disparity.

We see from the foregoing example, in which the resolution of the forces acting upon a kite have been worked out in detail, that the diagram of forces is a most powerful means of analysis. It has been the aim in the Weather Bureau investigations to exhaustively analyze the action of kites in the manner outlined above and thereby arrive at the best possible forms and proportions. With the limited time and means available for constructing kites and for preparing the apparatus and accessories required in making the observation, only partial solutions have thus far been reached, although the most gratifying improvements upon the original forms have even thus been effected. The line of study and experiment described above is better calculated to lead to improvements in kite flying than the simple flying of kites to just as great elevations as they can attain carrying meteorological instruments with them at the same time, so as to obtain atmospheric records. It is impossible by this latter method to analyze the action of the kite or to discover any except the most tangible and conspicuous imperfections. All the finer details leading to the development of the best forms and proportions of kites must always remain beyond the grasp of such experiments. Table IX contains the results of the efficiency tests made upon kites up to July 1, 1896.

TABLE IX.—*Results of efficiency tests.*

Date.	Kite. No.*	Kite. Kind.	Number of observations.	Amount of wire out.	Angular elevation of kite.	Inclinat'n of wire. At reel.	Inclinat'n of wire. At kite.	Incidence at kite.	Efficiency angle.	Efficiency.	Pull.
					Φ	θ'	θ	i	D		t
1896.				*Feet.*	°	°	°	°	°	%	lb
March 26	22	Three planes.	10	1,000	52.6	48.1	56.2	21.0	77.2	86	.....
April 28	24	....do.........	10	1,000	41.2	31.9	48.8	25.6	74.4	83	.....
May 11	24	....do.........	4	400	56.1	53.8	58.4	23.8	82.2	91	26
May 11	24	....do.........	10	400	57.6	54.0	60.5	23.8	84.3	94	27
May 15	24	....do.........	5	1,000	59.0	55.6	61.6	15.8	77.4	86	12
May 15	24	....do.........	10	1,000	61.0	57.1	64.2	16.0	80.2	89	15
April 22	23	Two planes..	6	400	55.3	51.1	58.7	23.9	82.8	92	.....
	23	....do.........	10	400	59.1	54.9	62.5	21.4	83.9	93	.....
April 21	28	Trapezoid....	12	290	56.0	53.7	58.3	16.3	74.6	83	.....
April 21	28	....do.........	10	1,000	49.6	44.8	53.5	17.6	71.1	79	.....
April 22	28	....do.........	5	400	57.7	55.0	60.4	17.8	78.2	87	.....
April 30	28	....do.........	9	1,000	53.4	48.9	57.1	14.9	72.0	80	.....
May 11	28	....do.........	10	400	56.4	54.5	58.3	15.6	73.9	82	27
June 11	28	....do.........	10	571	48.2	45.3	50.5	19.0	69.5	77	28.2
June 11	28	....do.........	10	571	50.5	47.2	53.2	15.1	68.3	76	22.9
May 11	19	Rect. struts..	14	400	46.6	41.9	50.5	17.9	68.4	76	8.6
	19	....do.........	10	400	53.9	48.5	58.3	17.9	76.2	85	8.4
April 28	29	Trapezoid....	10	1,000	45.0	36.7	51.7	18.3	70.0	78	.....
June 11	29	....do.........	10	571	53.5	50.0	56.3	15.5	71.8	80	31.1
June 11	33	Rect. struts..	7	571	48.8	44.0	52.7	22.8	75.5	84	10.7
June 11	35	Trapezoid ...	5	....	46.7	40.0	53.4	14.7	68.1	76	3
May 19	36	Rectangle....	12	700	58.0	54.8	60.6	20.1	80.7	90	17.8
	36	....do.........	10	2,000	53.8	45.6	60.0	22.2	82.2	91	20.0
	36	....do.........	10	2,000	52.6	42,4	60.2	22.2	82.4	92	12.2

* The kites in this table have the same numbers, respectively, as the corresponding kites in Table VI.

Bridle adjustment.—The adjustment of the bridle of the kite is not a matter of so much mystery and importance as is often supposed to be the case. It will be found, if proper experiments are made, that very much the same results can be obtained by the greatest variety of bridle arrangements, or even by discarding the bridle altogether. In the case of the kite shown in Fig. 70, exactly the same results would have been obtained if the bridle had been discarded and the wire attached directly to the kite frame at the point S. This, at least would be the case if there were no fluctuations in the wind, and its force and character had corresponded to the average of the observed variable wind. Likewise any one of many other forms of bridles, such as suggested by the several dotted lines in the diagram, might have been employed. The only condition which each of these arrangements must satisfiy is that the point of attachment of the wire must fall upon the line $L\ O$.

Steadiness in position.—We have said there would be no differences arising from the use of any of the several arrangements of bridles suggested, provided there were no fluctuations in the wind. We may go still further and say that although the extreme positions of the kite corresponding to variations of the wind might differ considerably, depending upon the bridle, yet it is quite probable that the averages would still be much the same. The complete analysis of this element of the kite problem is comparatively complex and a few important points only will be brought out here.

In the first place we have to deal with a highly complex set of variations of the wind. It will answer in the present discussion to consider only variations affecting considerable masses of air, such that the whole kite is subjected to uniformly changed conditions that persist long enough, at least, to permit the kite to assume a new position of equilibium. These variations may be divided into two groups: (*a*) variations of direction, (*b*) variations of force. In treating of the variations under (*a*) we must consider not only the incessant changes in horizontal direction, but must also recognize and deal with similar changes that are likewise going on in an up and down sense. The motions of considerable masses of air may be either upwardly or downwardly inclined as well as horizontal.

The variations of force are of great complexity, but their general character is pretty well known to every observer and need not be detailed here.

Changes of horizontal direction.—The changes in horizontal direction of the wind cause the kite to shift from side to side. So long as we tie the bridle only to the midrib of the kite, as is nearly always done, at least with the malay and cellular kites, all sidewise tiltings of the kite must take place about that stick as the axis. It does not matter, therefore, so far as these tiltings are concerned, how the arrangement of the bridle may be changed in other respects. In their direct effect on the sidewise movements of the kite all bridles are the same so long as they are fastened to the same stick or midrib.

Variations of force and direction.—Variations of either force or direction of motion, if inclined upward or downward, tend to cause the kite to rise or fall. If the variation is only of the inclination of the direction of motion of the wind, then the new position of equilibrium for the kite flying on a short and straight string will differ from the old by an angular amount (if measured from the reel) sensibly equal to the change in the inclination of the wind's motion. These angular changes would be exactly equal if it were not for a secondary effect, due to the weight of the kite, that need not be now considered. For such variations of direction as just considered the arrangement of the bridle in a particular case can not have any direct influence on the behavior of the kite.

If the variation is one of wind force, then the bridle adjustment may have much to do with the amount by which the kite will change its position. When the force of the wind is considerable, variations of the force will cause but slight changes in position of the kite, however bridled. When the force is only moderate, variations thereof produce larger changes in the position of the kite, and in such cases the following statements set forth rather crudely certain results depending upon the bridle. When the bridle is short, that is, when the point of attachment of the main line is relatively close to the surface of the kite, the angular changes in the position of the kite depending upon variations of wind force will tend to be greater than when the bridle is longer. Discarding the bridle, which can be done in cellular kites, gives a minimum distance between the point of attachment and the front surfaces, and is apt to result in large changes in angular elevation of the kite when the force of the wind falls off greatly. With short bridles, the angle of incidence of the kite tends to be more nearly constant with different wind velocities. Being nearly constant, the variations of pressure upon the kite will be nearly as great as those of the wind; whereas, the longer bridle permits the angle of incidence to increase when the velocity of the wind diminishes, in consequence of which the variations in the pressure upon the kite are less than the variations in wind force.

A very long bridle may produce conditions under which it is impossible for the kite to be in equilibrium.

The writer is accumulating numerical data by which the most useful proportions and disposition of the bridle in a given case can be fully established. As yet these studies have not been sufficiently advanced to justify more detailed statements than given above.

With a given form of bridle (preferably one in which neither of the angles next to the kite is a right angle), the angle of incidence of the kite will be made *smaller* if the point of attachment of the main line be shifted toward the *forward* end of the kite, and *vice versa*.

Lofty ascensions.—The favorable conditions of wind have been generally employed for the purpose of conducting those analytical studies of kite behavior which we believe to be the most helpful in developing the kite; yet efforts have been made, from time to time, to reach great elevations, either with a single kite or a tandem of two or more. Opportunities with favorable winds are, however, infrequent in Washington. Detailed observations of a few of the more successful high ascensions will give an idea as to what kites of the kind employed may be expected to do. These results are grouped in Table X.

TABLE X.—*Details of special ascensions.*

Date.	Time.	Angular elevation of kite. Φ'	Inclination of wire at reel. Θ'	Length of wire out. l	Approximate height. $l \sin \Phi'$	Remarks.
1896.	*h. m. s.*	° ′	° ′	*Feet.*	*Feet.*	
Jan. 27	....	43 00	38	1,300	886	Single diamond kite No. 5; 29 square feet surface; wind favorable at first, but gradually died out; pull from 20 to 24 pounds.
	....	35 30	17	3,490	2,027	
	....	28 47	....	4,834	2,328	
	....	28 45	....	4,894	2,354	
	....	34 15	....	3,767	2,120	
	....	38 38	....	2.273	1,419	
Feb. 10	....	34 7	....	3,844	2,156	Tandem of No. 9, 16.8 square feet; 200 feet below, No. 12, 12 square feet; 200 feet still lower, No. 5, 29 square feet; fair wind; total surface, 57.8 square feet.
	....	37 20	....	3,844	2,331	
	....	39 45	....	4,696	3,003	
	....	34 10	....	5,782	3,247	
	....	29 55	....	7,262	3,622	
Mar. 26	....	52 36	....	1,000	794	The first reading is the mean of ten made for measuring the incidence of kite, = 21.0°. These are the first incidence measurements made in the Weather Bureau experiments. Single kite; three-plane rectangular cells, No. 22, 38.4 square feet; wind very favorable; pull from 8 to 16 pounds with 3,975 feet out, and from 20 to 26 pounds with 6,010 feet out, showing considerable increase of velocity with elevation; inclination of wire at reel not recorded, but exceeded 10°.
	....	36 15	....	3,975	2,350	
	....	34 00	....	"	2,223	
	....	41 32	....	"	2,635	
	....	43 10	....	"	2,719	
	....	42 35	....	"	2,690	
	....	33 30	....	6,010	3,317	
	....	30 40	....	"	3,065	
	....	30 15	....	"	3,028	
	....	30 40	....	"	3,065	
	....	31 35	....	"	3,148	
	....	34 28	....	"	3,400	
	....	36 5	....	"	3,540	
	....	36 45	....	"	3,596	
	....	33 50	....	"	3,346	
	....	34 20	....	"	3,390	
	....	34 40	....	"	3,418	
	P. M.					
Apr. 30	1 29 30	53 24	48 54	1,000	803	
	1 36 15	45 42	....	1,000	715	
	2 15 00	61....	....	2,000	1,749	
	15 30	59....	....	"	1,714	
	16 00	57....	....	"	1,677	
	16 30	57 30	52....	"	1,687	The first and second observations are the means of ten readings made upon trapezoidal kite No. 28; 43.1 square feet of surface; the incidence was 14.9°; 700 feet from the first a second trapezoid, No. 29, 36.7 square feet, was attached on 150 feet of line; total surface, 79.8 square feet. The wind was not very favorable during these experiments, and it was with difficulty the second kite was started.
	17 00	58 30	52 30	"	1,705	
	17 30	56....	50....	"	1,658	
	18 00	55....	50....	"	1,638	
	18 30	55...	50....	"	1,638	
	19 00	54 30	49....	"	1,628	
	19 30	56....	48....	"	1,658	
	2 42 30	43 48	30....	4,000	2,769	
	44 10	46 50	31....	"	2,918	
	44 45	45 30	32....	"	2,853	
	45 20	45 59	30....	"	2,876	
	45 50	45 48	31....	"	2,868	
	46 35	46 5	30 30	"	2,880	
	....	46 47	28 30	"	2,915	
	47 10	46 23	26 30	"	2,896	
	49 21	45 21	21....	"	2,846	
	49 50	43 25	20....	"	2,749	
	50 48	40 41	21....	"	2,608	
	51 30	42 23	31 30	"	2,696	
	52 20	45 5	31 30	"	2,832	
	3 00 45	38 30	....	5,000	3,112	
	3 5	39 25	22 30	"	3,175	
	3 45	39 45	23....	"	3,197	
	4 20	40 5	22....	"	3,220	
	5 10	40 34	25....	"	3,252	
	5 48	41 8	26....	"	3,290	

TABLE X.—*Details of Special ascensions.*—Continued.

Date.	Time.	Angular elevation of kite. Φ'	Inclination of wire at reel. Θ'	Length of wire out. l	Approximate height. $l \sin \Phi'$	Remarks.
1896.	*h. m. s.*	° ′	° ′	*Feet.*	*Feet.*	
	6 35	41 57	25 ...	"	3,342	
	7 10	41 25	20....	"	3,308	
	8 5	39 30	21....	"	3,180	
	8 50	39 9	22....	"	3,157	
	3 17 40	33 40	17....	6,000	3,326	
	18 45	35 50	16 30	"	3,512	
	19 30	36 32	15....	"	3,572	
	20 5	36 59	16....	"	3,610	
	20 45	37 30	16 30	"	3,653	
	....	38 12	16....	"	3,710	
	22 20	38 30	16....	"	3,735	
May 28	2 34 00	41 35	....	6,430	4,268	Tandem of two kites. Three-plane kite No. 24, 38.4 square feet surface; two thousand feet lower down the trapezoid, No. 28, was attached, 43.1 square feet. The wind was just about right. The sky was partly overcast with clouds, and towards 3 p. m. it became apparent that a thunderstorm was likely to come up. The electrical discharges from the wire were very sharp, and followed each other in rapid succession, producing sparks an inch or more long. Means were not available at the time for measuring the pull, and the inclination of the wire could not be measured with the device usually employed, owing to the unpleasant effects from the electric discharges.*
	34 30	42 00	....	"	4,303	
	35 00	42 5	....	"	4,310	
	....	43 20	....	"	4,413	
	36 30	44 10	....	"	4,480	
	37 30	44 15	30 +	"	4,487	
	2 43 20	42 45	....	7,236	4,912	
	44 20	39 15	....	"	4,578	
	45	37 00	....	"	4,355	
	46	37 20	....	"	4,388	
	46 45	38 55	....	"	4,546	
	49	43 15	....	"	4,958	
	....	43 42	....	"	5,000	
	....	44 30	....	"	5,072	
	2 50 30	45 32	....	"	5,164	
	....	46 36	....	"	5,258	
	....	48 7	....	"	5,387	
	3 2 00	30 50	....	9,219	4,725	
	....	29 28	....	"	4,535	
	....	29 40	....	"	4,563	
	....	30 45	....	"	4,714	
	....	34 55	....	"	5,277	
	....	37 55	....	"	5,665	
	....	37 48	....	"	5,650	
	....	45....	....	9,000	6,364	
	....	45 30	....	"	6,419	
	3 15....	46....	....	"	6,474	
	....	46 45	....	"	6,555	
	....	46 00	....	"	6,474	

* The group of observations made with 9.000 feet of wire out represent the height of the base of the gathering clouds within which the kite was frequently obscured. About half past three p. m. a very severe thunderstorm burst upon us, and we were obliged to seek shelter. The kites continued to fly for several minutes during the storm, but finally broke loose. The storm was one of the most violent that has ever been known in Washington, and much damage was done throughout the city to roofs of houses, etc. A lofty steel flagstaff at Fort Myer, near the point at which the kites were flown, was bent over by the force of the wind at an angle of about 45° at the point about 50 feet above the ground, where it was held by guys. The kites were both found the same afternoon at a distance of 15 miles due east of the point from which they were flown. Neither kite had been damaged by the storm, and both are still in good condition.

THE KITE LINE.

Thus far in the study of the behavior of kites and in the analysis of the forces acting thereon we have considered, with few exceptions, only the kite itself. We now wish to study the forces acting upon the wire, with a view to clearly setting forth in what manner and to what extent these forces influence the elevation attainable with a given kite.

If we could employ a wire having no weight, and so fine that the pressure of the wind upon it would be wholly inappreciable, then, as more and more of this wire is paid out to it, the kite would pass outward and upward along the same straight line, such as $R\,K$, Fig. 72, retaining always the same angular elevation as seen from the reel. Provided the wind continued unchanged in force, there would be no limit to the height to which a kite could be flown under such circumstances. Unfortunately, however, we can not fly kites with wire having no weight and against which the wind will not press, and, in consequence, our actual kite behaves in a very different manner from that described above. Supposing, as before, that the wind force is the same at all points, high or low, the results we will actually obtain with the kite above employed will be something like these: When but a short length of wire is paid out to the kite, it will take its position upon the same line, $R\,K$, as before; that is, for example, at K_1. When more wire is unreeled, the kite does not continue upward on this line, but, instead, drifts gradually away to leeward and assumes, successively, such positions as at K_2, K_3, K_4, etc., which positions lie on a curve identical with that of the line, but having the ends and sags reversed. An important feature, common to all of the positions the kite may assume, is that *the portion of the wire next the kite remains always at exactly the same inclination.* The inclination is not only the same for all positions, but is the same as it originally was at $R\,K_1$. Changes of the wind force and other influences may cause this inclination of the wire to change, but the mere reeling out or in of the wire itself has no effect on the inclination. With a certain amount of wire out, the portion next the reel becomes horizontal, and the limit of altitude is then reached. The kite can lift no more line. All these effects have been brought about under the limitations imposed by the action of gravity and the wind upon the wire. We have mentioned the wind equally with gravity as affecting the wire. It is probable that with moderate wind forces the pressure upon wire, owing to its fineness in proportion to its weight and strength, is a smaller and less important force than gravity.

By the aid of well-known mathematical formulæ we can determine in the most complete and exact manner all the effects due to the action of gravity on the wire. On the other hand, the effects of the combined action of wind and gravity are of a very complex character, are but little known and understood, and can be mathematically represented only in a most general and imperfect manner. The effect of the wind pressure on the wire will be disregarded for the present and we will proceed to develop the properties of the curve assumed by the kite wire as if it were wholly dependent upon gravity alone. We will indicate afterwards how certain allowances can be made for the wind effect.

PROPERTIES OF THE CATENARY.

The name catenary is applied by mathematicians to the curve assumed by a chain or perfectly flexible inextensible string of uniform weight, when suspended from two points and acted upon by gravity alone. The kite wire is far from being perfectly flexible, but when the curve it assumes is formed on a large radius, as in kite flying, the wire may be regarded as perfectly flexible and the curve a true catenary, except for the wind effects. We may conceive that, owing to the stiffness and springiness of the wire, the curve in its minutest details acquires very small, but relatively long, waves and sinuosities. These, however, are utterly inappreciable and of no importance when steel wire is used. In the case of strings, the wind effect is more important, and, moreover, the extensible properties of the string prevent the actual curve from being a true catenary. We make mention of these disturbing influences, but do not attempt to give them further consideration.

The catenary possesses many very remarkable and interesting properties that have a more or less important bearing upon the art of flying kites. In presenting and treating of these properties we can scarcely avoid the use of certain equations, but we hope the verbal statements of results and conclusions reached by their aid will be interesting to both mathematical and non-mathematical readers alike.

The fundamental equations of the catenary may be written in a variety of forms, depending upon the variables employed. Each equation expresses some interesting property of the curve. Some of the forms most convenient for use are the following:

$$y = \sqrt{s^2 + c^2} - c \tag{1}$$

$$s^2 = y^2 + 2\,y\,c \tag{2}$$

$$x = c \text{ nap. log. } \frac{s + \sqrt{s^2 + c^2}}{c} \tag{3}$$

$$\tan.\,\theta = \frac{s}{c} = \frac{d\,y}{d\,x} \tag{4}$$

$$t = w\,(c + y) \tag{5}$$

In these equations the origin of coordinates is taken at the point where the curve is horizontal; s is the length of the curve measured from the origin, c is a constant, θ is the angle of inclination of the curve with the horizontal at the upper end of a portion of length s, t is the tension at this upper end, and w is the weight per unit length of the material of which the catenary is formed.

In Fig. 73 let $A\ O\ B$ represent a catenary. The curve has similar branches on either side of $O\ Y$, but we are generally concerned with only a portion of the curve on one side. If the wire is just horizontal at the reel, then the position of the reel will be represented by the point O in the diagram. If the wire at the reel is inclined upward, more or less, then the position of the reel will be represented on the diagram by some such point as R, at which point the curve is inclined at the same angle as the wire at the reel.

Tension.—The tension of the wire at the lowest point, that is at O, when the curve is horizontal is less than at any other point. The quantity c in the equation above is given by the expression $c = \frac{t_0}{w}$. That is, c is the length of a piece of wire whose weight equals t_0, the tension in the curve at the lowest point. Extend the line $Y\ O$ down to O', making $O\ O' = c$, and draw the horizontal line $D\ D'$. This line is known as the *directrix* of the catenary. We found above that c was the length of a piece of wire whose weight equaled the tension at the lowest point. Any other vertical line, such as c', drawn from a point p on the catenary to the directrix represents, in like manner, the tension at the point p.

If $t\ \theta$ and $t'\ \theta'$ are, respectively, the tensions and inclinations of the curve at any two points, then, from equations (1), (4), and (5), there results,

$$\frac{t}{t'} = \frac{\cos.\,\theta'}{\cos.\,\theta} \tag{6}$$

Maximum height.—Let P represent the point at which the kite acts on the wire, and suppose that the reel is at O, the kite will then be at its maximum height, which is represented by the ordinate y. The whole catenary is sustained by the pull of the kite. This pull is exerted in a certain direction, and with a certain intensity. It was pointed out above that with a steady, constant wind force, and the same kite, the direction and intensity of the pull remains fixed and invariable. Let the inclination of the wire next the kite be represented by the angle θ, as indicated in Fig. 73; then, as seen from the reel, the kite will have the angular elevation $P\ O\ X = \Phi$. If s is the length of the wire up to the kite then the height of the kite will be, from equation (1),

$$h = y = \sqrt{s^2 + c^2} - c$$

Replacing c in this equation by its value in terms of $\tan.\,\theta$, and reducing, we obtain

$$h = y = \frac{s}{\sin.\,\theta}(1 - \cos.\,\theta) \tag{7}$$

This equation tells us that when a kite has taken up all the line it can carry the height may be expressed in terms of the length of the line and the inclination of its topmost portion. If we imagine several kites in the air, some small ones restrained with fine threads and strings, others larger with fine wires, others again still larger with heavy cables, and if we suppose further that all these kites pull their respective lines at the same angle θ, and that when the same length of line is out the bottom end is just horizontal, then equation (7) tells us that all these kites will be at the same elevation and that the curves of their respective lines will be exactly alike whether the lines are light or heavy. The only difference in the conditions existing in the several lines will be one of tension, which will necessarily be greater in the heavy than in the light lines. These statements are graphically verified by a very simple experiment. Take several chains or other very flexible strings of very different weights per lineal foot, suspend exactly equal lengths of these chains and strings between any two points, the curves assumed will be identical. We learn further from equation (7) that so far as the action of gravity on the kite line is concerned nothing is to be gained or lost by the use of either light or heavy lines. The tension under given conditions will be exactly proportional to the weight of the line employed. Heavy lines will require proportionally larger kites to produce the same effects. This is evident from equation (5)

$$t = w\,(c + y) = w\left(\frac{s}{\tan.\,\theta} + h\right) \tag{8}$$

in which for the same values of s, θ, and h the tension is directly proportional to w.

Angular elevation at maximum height.—Returning to the consideration of a single kite at P, Fig. 73, Φ is the angular elevation of the kite observed at the horizontal point of the curve and when the linear altitude of the kite is a maximum. From trigonometry we have

$$\tan.\,\Phi = \frac{y}{x}$$

Substituting in this equation the value of y in equation (7) and x from equation (3), eliminating c by means of equation (4), and reducing, we get

$$\tan.\,\Phi = \frac{y}{x} = \frac{1 - \cos.\,\theta}{\cos.\,\theta\ \text{nap. log.}\,(\sec.\,\theta + \tan.\,\theta.)} \tag{9}$$

The second member of this very interesting equation contains only the quantity θ. The meaning of this is that when a kite has taken out all the line it can carry, or when the line at the reel is horizontal, the kite's angular elevation will be a minimum, and will depend entirely upon the inclination of the upper part of the line next the kite. If we imagine several kites of different sizes pulling with different forces, but all pulling their respective lines at the same angle, then these kites, when each has lifted all the wire it can carry, will all have the same angular elevation measured from the lowest point of the line. If these lowest points are all brought together at a common point represented, for example, at O, in Fig. 73, the kites will all take up positions one behind the other as at P, P', P'', etc., on the straight line, $O\ P$.

Isoclinals.—It results from the above that if we draw a large series of catenaries, each corresponding to a given value of c, upon the same coordinate axes as in Fig. 74, then a line like $O\ C$, radiating from the origin O, will intersect every conceivable catenary at the same angle, and the tangents to the curves at the points of intersection will form a system of parallel lines. Any other radial line, as $O\ C'$, will intersect at a new angle and form a different set of parallel tangents. The radial lines under these circumstances may be called isoclinals, and designated C_{50}, C_{60}, etc., corresponding to the angles of inclination of the curve at the points of intersection. All conceivable catenaries formed upon the coordinate axes $O\ X$ and $O\ Y$ must, in the diagram, be comprised within the space above the axis $O\ X$ and no two of the catenaries can intersect. Fig. 75 is a diagram embracing a comprehensive system of lines, catenaries, etc., formed upon the principles stated above. These principles have important applications with respect to the behavior of kites.

The angle of inclination of that part of the wire that is next to the kite, or the bridle, tends, as we have seen, to remain comparatively constant, it changes to some small extent with changes in the force and vertical component of the wind, and the angle differs more or less in different kites. Other things remaining the same, however, the real problem in designing kites that shall attain great elevations is to cause this angle to be as great as possible. We see now the reason for this. The position of a kite which pulls the wire at an angle of 50° to the horizontal must, for the maximum height, be represented by a point on the line $O\ C_{50}$ of Fig. 75. The corresponding angular elevation Φ, as seen from the reel and as given by equation (9), is only $\Phi = 28° 48'$, and it makes no difference what kind of line is employed or how much is paid out, the position of the kite pulling at an angle of 50° must, when it attains its maximum elevation, be represented by a point on the isoclinal C_{50}. Similarly, a kite pulling at 60° attains its maximum elevation at an apparent angular altitude of $\Phi = 37° 13'$, and in the diagram, Fig. 75, its position is represented by some point on the isoclinal C_{60}.

Isoclinals for practical cases.—Having thus, from the properties of the catenary, learned the effects resulting from pulling the upper end of a kite line in different directions, let us refer to actual observations on kites and ascertain at what angles the wire is actually pulled in practical cases. Table IX contains the results of numerous observations upon kites and the angles we now seek are given in one column under the heading θ = inclination of wire at kite. The smallest θ angle recorded is 48.8° and the largest 64.2° and it happens that both results were obtained with the same kite, namely, the three plane kite shown in Fig. 56.[1] The difference between these two values is partly due to differences of wind force, but also to alterations made in the bridle on different occasions. Our experience with this class of kites shows that the angle between the horizontal and the wire next the kite rarely exceeds 60°, except with kites of the best form and under very favorable conditions of wind. A greater inclination than 60° may in some cases be obtained with kites of light weight by adjusting the bridle so that the angle of incidence is small. In that case, however, the wind pressure is lessened and the gain that arises from a steeper angle of pull is more than counterbalanced, perhaps, by the diminution in the amount of the pull. The selection of the most advantagous angle of incidence is an interesting point which will be considered later.

Equitensals.—Referring again to Fig. 75 we recall that we found that, when at their maximum height, the positions of all kites pulling at 50° may be represented by points on the isoclinal C_{50}, similarly those of kites pulling at 60° by points upon the isoclinal C_{60}. Now, suppose it were possible to cause a kite to continue to pull with the same constant force, while the direction of the pull at the kite is changed, it will be interesting to inquire what effect a change in the angle of pull can produce upon the maximum possible elevation of a kite. From a mathematical standpoint the answer to this question consists in drawing a line in Fig. 75 of such a character that the tensions on all the catenaries at the points of intersection with the new line will be the same. Such intersecting lines may be called equitensals, since they cut the catenaries at points of equal tenseness or pull on the line. We may find the equation of such a line as follows: From equation (8) we have for the tension at a point whose elevation is h and where the curve is inclined at an angle θ,

$$t = w\left(\frac{s}{\tan.\theta} + h\right)$$

from which

$$\frac{s}{\sin.\theta} = \frac{t - h\,w}{w\cos.\theta}$$

[1] Fig. 56 will be found in the WEATHER REVIEW for May.

Substituting this expression in equation (7) and solving for h we have

$$h = \frac{t\,(1 - \cos.\theta)}{w} \qquad (10)$$

which is the equation sought. This equation may be stated in another form, in terms of s, by deriving it in a similar manner from equations (7) and (8) by eliminating h. The result is

$$s = \frac{t}{w}\sin.\theta \qquad (11)$$

Equation (10) gives us the maximum height attainable by a given kite pulling at an angle θ with tension t, the wire weighing w pounds per unit length. Equation (11) gives the length of wire required by the kite to attain this position.

In Fig. 75 $T\ T'$ is an equitensal passing through the point P. The points at which this line crosses the isoclinals C_{50}, C_{60}, C_{65}, etc., are the positions that would be taken by kites that are at their maximum altitudes and all pulling equally hard, but at angles of 50°, 60°, and 65° respectively. In constructing any equitensal, such as $T\ T'$, we observe that if h_{50} equals the height at which the equitensal crosses the isoclinal of 50°, then the height at which it crosses the isoclinal of 60° will be

$$h_{60} = h_{50}\frac{1 - \cos.60°}{1 - \cos.50°} = 1.400 h_{50}$$

Drawing a horizontal line on the diagram at a height $= 1.4 h_{50}$ above the line $O\ X$, the point at which it intersects the isoclinal C_{60} is a point on the desired equitensal. Other points may be located in a similar manner.

Furthermore, equation (11) shows that if s_{50} is the maximum length of the curved line of wire that a kite pulling with a certain force can sustain when the angle of pull at the kite is 50°, then by pulling with the same force at an angle of 60°, it will carry up a length of wire given by the expression

$$s_{60} = s_{50}\frac{\sin.60°}{\sin.50°} = 1.130 s_{50}$$

These results may be presented in another and perhaps more striking manner. Suppose a kite pulling with a certain force at an angle of 50° is able to attain a maximum elevation of 1,000 feet. If now, by any means, we can cause the kite to pull with the same force at an angle of 60° instead, it will attain an elevation of 1,400 feet, being a direct gain of 400 feet in 1,000 for an increase of 10° in the angle. The length of wire required in the first case will be 2,145 feet, and in the second case 2,425 feet. Although 400 feet have been gained in elevation by the change, yet only 280 feet more of wire have been required. With the kind of wire employed in the Weather Bureau work, weighing 2.155 pounds per 1,000 feet, the tension required at the kite in both cases will be 6.03 pounds. The weight of the additional 280 feet of wire is 0.603 pounds. The kite then, without pulling any harder, flies 400 feet higher and carries 0.603 pounds more wire. This gain in height and carrying power is wholly due to the improvement in the angle of pull in the kite. It is important to notice here that this increase in the angle of pull must not be brought about, as it might be, by lessening the angle of incidence of the kite, because in that case the pull of the kite would also be lessened, and our comparison has been drawn on the supposition that the pull has remained constant. There is a way, however, in practical cases by which the desired improvement in the *direction* of the pull can be brought about without sensibly diminishing the *intensity* of the pull. If the kite pulling at 50° is badly defective in respect to edge pressures, waviness and fluttering, eddy effects, etc., then by eliminating these defects the angle of pull will be increased with only a very a very slight diminution of pull. From actual measurements upon Weather Bureau kites, gains of as much as 10° in the angle of pull are sometimes possible in practical cases with no loss in intensity of pull.

Incidence for maximum altitude.—We have noticed before that the advantage which may be gained by lessening the angle of incidence of the kite, and which, other things remaining the same, would tend to make the direction of pull steeper, may be more than counterbalanced by the diminution in the intensity of the pull, which necessarily accompanies a diminution of the angle of incidence. Furthermore, there is another wholly independent and very important factor bearing directly upon this question, namely, the efficiency as affected by changes in the pressure of the wind. It was shown on page 241 that when the wind pressures upon kites became relatively small, as may be the case with relatively small angles of incidence, the efficiency angle, owing to the pronounced effect of the weight of the kite, also became small. We may state this in other words, as follows: Lessening the angle of incidence not only always lessens the pull but it may also lessen the angle at which the kite pulls the string, owing to the detrimental effect of the weight of the kite under feeble wind forces. If we set the kite at too great an angle of incidence it will fail to reach a great elevation, because in spite of the strong pull it may exert, the direction of this pull is at too unfavorable an angle for the best effect. On the other hand, too small an angle of incidence, owing to the falling off in efficiency, likewise fails to bring about the most satisfactory result. It is apparent, however, that between these extremes is a condition, a particular angle of incidence, leading to the maximum linear elevation. On account of the change which may take place in the efficient action of the kite when the incidence of the kite is changed, and arising more particularly in light winds it is probable that the *incidence for maximum effect* should be determined independently and separately for each kite. Data is not available by which this can be done at present, and it will be quite as instructive, in the present case, to analyze the problem in a general way. This will give an idea as to the approximately best angle of incidence.

Ideal and actual kite.—There are two conditions for which we may seek the solution of this problem. We may consider only the special case of the ideal kite, with a constant efficiency of 100 per cent, or we may ascertain the best incidence of actual kites of several stated efficiencies. The complete solution would require that we suppose the efficiency to vary as a function of the incidence. It is in respect to this condition that data is as yet wanting. We will, therefore, first solve the equations for the ideal conditions, and afterward consider the actual kite, with several different efficiencies, in order to give a range between which most practical cases will fall.

Best incidence—ideal case.—If i is the angle of incidence of the kite, then, in the ideal case, the direction of pull will be,

$$\theta = (90^\circ - i).$$

Now, the force with which the wind presses upon flat surfaces at different angles of incidence is given with a close degree of approximation by Duchemin's formula, as follows:

$$P = P_0 \frac{2 \sin. i}{1 + \sin.^2 i} \qquad (12)$$

In this expression P represents the proportional pressure upon the inclined surfaces of the kite and P_0 the corresponding pressure of the wind upon the same surfaces exposed normally to the wind direction. The formula is strictly applicable to flat surfaces only. It is applied to kites in the manner that follows because a better formula is not known. We desire to know, at least approximately, which is the best angle of incidence in a given case, and this we believe Duchemin's formula will give.

The pull of an actual kite—that is, the tension in the wire at its upper end—is represented by the diagonal of a parallelogram, of which P from the above equation is one side and W, the weight of the kite, is the adjacent side. The included angle is $180^\circ - i$. In the ideal kite we assume that the weight is inappreciable, compared with the wind force on the kite, and, as a direct consequence of this assumption, the diagonal of the above mentioned parallelogram coincides with the side P; in other words, in the ideal kite the pull is equal to the pressure of the wind; hence we may write for the tension in the wire at the upper end,

$$t = P = P_0 \frac{2 \sin. i}{1 + \sin.^2 i}$$

From this equation and (10), first replacing θ in the latter by its value, $\theta = (90^\circ - i)$, we have:

$$h = \frac{2P_0}{w} \frac{\sin. i - \sin.^2 i}{1 + \sin.^2 i} \qquad (13)$$

This equation gives in terms of the angle of incidence the height attainable by a given ideal flat kite when it has taken out all the line it can sustain. To find the incidence which will give the maximum possible elevation, we need only to determine the value of i from the differential coefficient of equation (13) when that coefficient is placed equal to zero. That is,

$$\frac{dh}{di} = \frac{2P_0 \cos. i}{w(1 + \sin.^2 i)^2} \left[1 - \sin.^2 i - 2 \sin. i \right] = 0 \qquad (14)$$

whence

$$\sin.^2 i + 2 \sin. i = 1. \qquad (15)$$

That is,

$$\sin. i = \pm \sqrt{2} - 1 = +0.4142 \text{ or } -2.4142$$

and

$$i = 24^\circ\ 28'.$$

The angle of incidence with which the ideal flat surface kite can attain the highest elevation is therefore 24° 28′, and the corresponding inclination of the wire at the kite is 65° 32′. The angular elevation of the kite from the reel when the wire is horizontal will be, from equation (9), $\Phi = 42^\circ\ 47'$.

Best incidence for actual kite.—In the case of the actual kite the efficiency will necessarily always be less than 100 per cent, which is practically equivalent to saying that in the actual kite the angle between the wire and the kite will always be less than 90°. This angle of the string is affected by: (1) the wind pressure upon the edges of the kite, waviness, fluttering, eddies, etc., which deflect the action line of the total wind pressure upon the kite away from normal, (2) the weight of the kite must be overcome, and to do this the direction of *pull* must be deflected away from the direction of the *wind pressure*. Both these effects (1) and (2) act in the same manner; that is, if g represents the angular deflection due to gravity or the weight of the kite, and e that due to edge pressures, then the direction of pull will be deflected away from the normal to the kite surfaces by an angular amount, represented by $(e + g)$. The relations of the angles in question are shown in Fig. 76. If P represents the pressure of the wind normal to the kite surfaces, then the total wind pressures $O\,Q$ will be $P' = P \sec. e$. Furthermore, in the triangle of forces $O\,Q\,R$, from trigonometry, the side $O\,R$ = pull of kite, will be given by the expression,

$$t = \sqrt{P^2 \sec.^2 e + W^2 - 2\,P\,W \sec. e \cos. (i + e)} \qquad (16)$$

The angle e is not a known quantity; it is a small angle which is, it seems, practically constant in a given kite, but may possibly vary with the wind force. This angle, in certain kites has been determined by means of the diagram of forces which is described on p. 243. The angle in the best cellular kites has been found to be under 3°, whereas with inferior kites the value has slightly exceeded 10°. The term sec. e is, therefore, on account of the small value of e, a quantity which we may assume to be constant without introducing any important error.

In regard to the term cos. $(i + e)$ it may be said that i, the best incidence for the actual kite must necessarily be smaller than that for the ideal flat surface kite, which we have found to be 24° 28′. The reason for this is that the effects due to edge pressures, waviness, eddies, etc., tend to depress the kite by forcing it to leeward away from the zenith. To offset this it is necessary to set the kite at a smaller incidence which tends to make it approach the zenith point. We may therefore expect to find the best incidence for the actual kite with flat surfaces smaller than 24°. Since e, as we have seen for the better class of cellular kites observed, is less than 3°, we may assume that $i + e$ will not exceed 25° in actual kites. Moreover the term can not change its value more than a few degrees in extreme cases, which fact together with the general unimportance of the term in any case renders refinement unnecessary and we will therefore assume that this term has the constant value,

$$\cos.\,(i + e) = a$$

In work with actual kites we can not profitably attain high elevations unless the wind force upon the kite is considerably greater than the weight of the kite. Under ordinarily favorable condition the wind force P will be from 5 to 7 times the weight of the kite and will frequently be still greater. As we seek more particularly to discover the best incidence under conditions of favorable winds we will assume that the weight of the kite in equation (16) is expressed in terms of P, thus, $W = b\,P$, in which b is a small fraction rarely as great as 0.2 and often less than 0.1.

According to the several assumptions we have made above equation (16) becomes,

$$\text{Pull} = t = P\sqrt{1 + b^2 - 2\,a\,b} = k\,P$$

and adopting Duchemin's formula, equation (12), as applicable to cellular kites with flat surfaces, we get,

$$t = k\,P = k\,P_0 \frac{2 \sin.\, i}{1 + \sin.^2 i} \tag{17}$$

In reducing the expression (16) to this form we virtually assume that the tension on the wire next the kite does not undergo any variations with changes of incidence except such as are wholly due to changes in the wind force. This is not strictly the case, for there is a slight variation due to the effects of the weight of the kite and these are fully included in (16). The amount of these variations, however, in the extreme cases will barely attain to 1% of the pressure itself, and we believe that by neglecting them, as we shall do, no serious error will result in the values deduced for the best angle of incidence.

From Fig. 76 we see that

$$\theta = 90° - (e + g) - i.$$

$90° - (e + g)$, it will be noted, is the angle of inclination of the wire to the kite and is a known angle when the efficiency of the kite is known. We have heretofore called this angle the efficiency angle (page 239). Knowing the percentage efficiency, E, of a kite, the efficiency angle, D, is given by the relation,

$$D = 90 \times E$$

and for the inclination of the wire at the kite we may write

$$\theta = D - i$$

with the values of t and θ, given above, and equation (10), we obtain the following equation for the maximum elevation that can be attained by actual flat surface kites depending upon the pull and the angle of incidence; (13) is the corresponding equation for ideal kites,

$$h = \frac{2\,k\,P_0}{w} \frac{(\sin.\, i - A \cos.\, i \sin.\, i - B \sin.^2 i)}{(1 + \sin.^2 i)} \tag{18}$$

In this equation $A = \cos.\, D$ and $B = \sin.\, D$ are sensibly constant for any given kite under conditions of wind force favorable for gaining high elevations.

When the efficiency is 100% $D = 90°$ and $k = 1$. Equation (18) then reduces to (13) for the ideal kite as should be the case.

Differentiating (18) and reducing, we have,

$$\frac{dh}{di} = \frac{2\,k\,P_0}{w\,(1 + \sin.^2 i)^2} \begin{bmatrix} (\cos.\, i - A) \cos.^2 i \\ + 2 \sin.\, i\,(A \sin.\, i - B \cos.\, i) \end{bmatrix} \tag{19}$$

which is quite analogous to the similar equation (14) for ideal kites. Placing the second member equal to zero for a maximum, we obtain a form convenient for computation, as follows:

$$\cos.\, i = A \left[1 - 2\left(\tan.^2 i - \frac{B}{A} \tan.\, i\right)\right] \tag{20}$$

B and A, it will be remembered, depend upon the efficiency. When this is 100 per cent, equation (20) reduces to,

$$\sin.\, i = \pm \sqrt{2} - 1,$$

the same as already found for the ideal kite.

By means of equation (20) the best angle of incidence for kites of several different degrees of efficiency, ranging from 70 to 95 per cent, have been computed by methods of approximation, and are given in Table XI, with other useful information. Efficiencies as low as 70 per cent ought not to obtain with good kites, except, perhaps, in very light winds, in which case ascensions to considerable elevations with such kites are not practicable. On the other hand, an efficiency of 95 per cent is not by any means unattainable when the wind velocity is favorable—that is, 15 miles per hour or more.

TABLE XI.—*Best angles of incidence for flat-surface kites.*

	Efficiency.						
	70%	75%	80%	85%	90%	95%	100%
Efficiency angle.. D	63° 00′	67° 30′	72° 00′	76° 30′	81° 00′	85° 30′	90° 00′
Best incidence.... i	18° 30′	19° 33′	20° 36′	21° 36′	22° 34′	23° 31′	24° 28′
Inclination....... Θ	44° 30′	47° 57′	51° 24′	54° 54′	58° 26′	61° 59′	65° 32′
Elevation......... Φ	24° 49′	27° 17′	29° 53′	32° 42′	35° 46′	39° 07′	42° 47′
Altitude, feet .... h	1,000	1,202	1,424	1,666	1,928	2,207	2,504
Pull, pounds ...... t	7.5	7.8	8.2	8.4	8.7	9.0	9.2
Length of wire.... s	2,444	2,703	2,959	3,207	3,447	3,674	3,890
Ratio.......... $h \div s$	0.410	0.444	0.481	0.518	0.559	0.602	0.645

In addition to the best angles of incidence for actual kites of several efficiencies, Table XI gives the maximum heights attainable, computed from equation (18), upon a uniform basis of such conditions as would be required by the kite of 70 per cent efficiency to attain an elevation of 1,000 feet; that is, if the efficiency of this same kite could be increased from 70 per cent to 90 per cent, for example, and with no change whatever in its surface, weight, or other features, it would then, with exactly the same wind, be capable of attaining nearly double the altitude, namely, 1,928 feet. The constant required in equation (18) for these computations is obtained by making $h = 1{,}000$ when $i = 18° 30′$, and solving for $2\,k\,P_0 \div w = 12{,}090$. The assumption that k is constant, as explained above, will not affect the results to an important extent. The pull, t, at the kite and the length of wire, s, may be found most easily from equations (10) and (11), respectively, in which w is the weight per foot of the steel wire employed at the Weather Bureau, viz, 0.002155 pounds.

A kite showing an efficiency of 85 per cent will, in most cases, be regarded as a very good kite, although still higher efficiencies up to 95 per cent are probably attainable. The altitude attained by an 85 per cent kite is less than that of the 95 per cent kite by 541 feet on a moderate elevation of 1,666 feet. For an ascension of 1 mile the 85 per cent kite would be deficient by over 1,700 feet, that is, the 95 per cent kite under precisely the same circumstances would ascend 1,700 feet more than the mile.

It is plain that where such large gains as this are possible, it devolves upon every one who aims to get the highest elevations to fully inform himself as to the real merit of his kites and see to it that they are bridled and flown under the best adjustments.

The results which have been brought out in the foregoing discussions concerning the best incidence depend upon Duchemin's law of variations of pressure with incidence, and apply only to kites with flat as distinguished from arched surfaces. The best incidence for arched surfaces is undoubtedly smaller than for flat surfaces. We have also disregarded the effect of the wind upon the wire, which while small, is still of some importance, and as its effect is to drift the kite to a position further away from the zenith than would otherwise be attained, the best incidence when the wind effect is included will be smaller than given in Table XI.

Maximum sag and slack of wire.—We have called the angles between the curve and its chord the sag of the wire, as for example the angles S and S', Fig. 67. We will similarly use the term *slack* to designate the difference between the length of the chord and the length of the curve itself.

When the wire is horizontal at the reel the angle of sag at that point is then the same as the angular elevation of the kite, that is $S' = \Phi$, the sag at the kite is similarly, $S = \theta - \Phi$. Dealing with portions of the catenary on one side only of the Y axis, S' is the maximum sag possible.

If r is the air-line distance between the reel and the kite when the wire is horizontal, then,

$$r = \frac{h}{\sin. \Phi}$$

combining this equation with (7) we get,

$$r = \frac{s\,(1 - \cos. \theta)}{\sin. \theta \ \sin. \Phi}$$

and the slack will be,

$$s - r = s\left(1 - \frac{1 - \cos. \theta}{\sin. \theta \ \sin. \Phi}\right)$$

We will consider hereafter the sag and slack for conditions less than the maximum.

Partial ascensions.—In the discussion of the properties of the catenary we have thus far treated only of the behavior of kites when they have ascended to their utmost limit and sustain all the wire they can carry. All those conditions which tend to produce the best results when the wire is horizontal at the reel are equally beneficial in the case of partial ascensions where the kite carries up only part of the wire it can sustain, and the portion at the reel is inclined to the horizontal at a slight angle. Partial ascensions are the usual cases in practice. When the wire at the reel becomes horizontal the frequent diminutions of wind force allow it to temporarily sag to the ground or to interfere with trees, buildings, etc., and in general, therefore, we must provide some margin within which the usual variations of pull may occur without permitting the wire to sag to an objectionable extent. Furthermore we see from Fig. 72 that, since the path described by the kite in attaining its maximum elevation is the inverted catenary, the last portion of the ascent is very slight, and but little is gained in paying out wire to the last extremity.

The constancy of the inclination of the upper portion of the wire in the successive positions assumed by a kite passing upward from the reel to a maximum elevation, as shown in Fig. 72, was pointed out on page 246. The several curves of the wire are all portions of one and the same catenary, that is, portions of the curve $R\,K_3$. When but a short length of wire is out, its curve is the portion of the catenary from K_3 down to such a point as R_1. With greater and greater lengths of wire out it is as if the reel were moved backward and downward along the catenary passing through positions such as R_2 R_3, etc., while the kite has remained stationary. When we know the angle of inclination of the wire at the reel in a given case we can locate its position on the catenary. The diagram in Fig. 75 represents all conceivable catenaries and may therefore be employed to represent graphically any partial ascension. For example, if the wire at the reel is inclined at an angle, $\theta' = 10^\circ$, then the position of the reel is represented in the diagram by some point on the isoclinal C_{10}. The particular point on the isoclinal will depend upon the tension, t', at the reel. If this is known, then the position of the reel is located at the point of intersection of the isoclinal C_{10} and the equitensal t'. The catenary passing through the point of intersection is the particular one representing the kite wire in the given case and the position of the kite at the upper end may be located in several ways.

If θ, the inclination of the wire at the kite is, for example, $\theta = 60^\circ$, then the position of the kite will be represented by the point of intersection of the particular catenary already found with the isoclinal C_{60}. If Φ' is the angular elevation of the kite from the reel we may lay off on the diagram a line making the angle Φ' with $O\,X$ and passing through the point representing the position of the reel. The upper intersection of this line, with the particular catenary representing the kite line, gives the position of the kite. There is still another and more general graphical way of locating the kite on the diagram. It is possible to draw a system of lines on the diagram resembling the equitensals and crossing the catenaries, but cutting off *equal arcs* of the curves measured from the origin. The equation for these *equiarcals* is obtained simply by making θ and h the variables in equation (7) thus:

$$h = \frac{s}{\sin. \theta}\,(1 - \cos. \theta)$$

Lines of this character are designated on the diagram by the letters L_1, L_2, etc. The subscripts indicate the length of arc cut off from the origin in units of 1,000 feet. Having located on the diagram the position of the reel, in the case of a partial ascension, the equiarcal passing through that point gives the length on the catenary from the reel to the origin. Knowing, in addition to this, the length of wire out, the sum of the two determines the equiarcal for the kite. The point of intersection of this with the particular catenary passing through the reel gives the desired position of the kite.

The linear elevation of the kite is the vertical distance on the scale of the diagram between the positions found for the reel and the kite.

By such methods as we have thus described a diagram of the kind shown in Fig. 75 may be employed as a graphic chart completely representative of any ascension that may be made with a single kite. Numerical tables for deducing elevations, etc., will probably be preferable in many cases but the chart shows the results graphically and has been discussed at length more particularly because of the several interesting properties of the catenary involved in its use.

General equations for partial ascensions.—Fig. 77 represents a partial ascension in which the reel is at R and the kite at

K, with the origin of coordinates at O. Letters designating the coordinates of the catenary at the point representing the reel are distinguished by a superscript, ($'$). The linear elevation of the kite is $h = y - y'$ and the length of wire out is $l = s - s'$.

If t' is the tension of the wire at the reel then from equation (10) we have,

$$y' = \frac{t'}{w}(1 - \cos.\ \theta')$$

Eliminating c from equation (1) by its value in terms of t' and θ' and replacing s by its value $s = l + \frac{t'}{w}\sin.\ \theta'$ we obtain,

$$y = \sqrt{l^2 + \frac{2\,l\,t'}{w}\sin.\theta' + \left(\frac{t'}{w}\right)^2} - \frac{t'}{w}\cos.\theta' \qquad (21)$$

Whence,

$$h = y - y' = \sqrt{l^2 + \frac{2\,l\,t'}{w}\sin.\theta' + \left(\frac{t'}{w}\right)^2} - \frac{t'}{w} = r\sin.\ \Phi' \qquad (22)$$

From this equation we learn that when the length of wire out is known together with the tension and inclination at the reel, the height of the kite is given, even though it is concealed from view, as by clouds, darkness, its remote distance, etc. This results from a general property of the catenary and the equation is equally applicable to the case of either partial or complete ascensions. Owing to great momentary variations that take place in the tension of the wire, calculations of elevations depending upon the tension at the reel will not, as a rule, be as accurate as those deduced by other methods, but equation (22) will undoubtedly prove useful in cases where other methods of ascertaining elevation are not available.

In passing, it may be remarked that the elevation of an invisible kite deduced by equation (22) will be more accurate, as the sag in the wire is greater.

If θ and t are the inclination and tension of the wire at the kite, we may write,

$$y = \frac{t}{w}(1 - \cos.\ \theta), \text{and } y' = \frac{t'}{w}(1 - \cos.\ \theta')$$

whence, by equation (6), we get,

$$h = y - y' = \frac{t}{w}\left(1 - \frac{\cos.\ \theta}{\cos.\ \theta'}\right) = r\sin.\ \Phi' \qquad (23)$$

an equation which we shall have occasion to use hereafter.

Observed angular elevation.—Instead of measuring the tension in the wire at the reel in a given case, we may observe the angular elevation, Φ', of the kite from the reel, and if we can determine the relation between Φ' and t', the latter may be eliminated from equation (22). From trigonometry we have

$$\tan.\ \Phi' = \frac{h}{x - x'}$$

The value of x' in terms of t' and θ', deduced from equations (3), (4), and (11), is,

$$x' = \frac{t'}{w}\cos.\ \theta' \text{ nap. log. } (\sec.\ \theta' + \tan.\ \theta') \qquad (24)$$

Similarly the value of x is,

$$x = \frac{t'}{w}\cos.\ \theta' \text{ nap. log. } \frac{l + \frac{t'}{w}\sin.\theta' + \sqrt{l^2 + \frac{2\,l\,t'}{w}\sin.\theta' + \frac{t'^2}{w^2}}}{\frac{t'}{w}\cos.\ \theta'}$$

From these values of x and x' and the value of h given in (22), we obtain a very complex transcendental equation, representing the relation between the angular elevation at the reel and other quantities that are known. The value of t' corresponding to a given value of Φ' can be deduced from this equation only by methods of approximation. It will not, therefore, be practicable to eliminate t' from equation (22) in the manner contemplated, but we can, by tabulating a limited number of values of the several quantities, deduce the percentage of slack in the wire corresponding to such conditions as are likely to occur in practice, and thus provide a method for accurately computing the height of kites, in partial ascensions, that does not depend upon the tension of the wire.

Slack in the wire in partial ascensions.—Let r be the length of the chord of the catenary from the reel to the kite, then,

$$r = \frac{h}{\sin.\ \Phi'} \qquad (25)$$

$$\text{slack} = l - r \text{ and percentage of slack} = 1 - \frac{r}{l}$$

The ratio of any chord of a catenary to the corresponding arc is given by the equation

$$\frac{r}{l} = \frac{\cos.\ \theta' - \cos.\ \theta}{\sin.\Phi'\sin.(\theta - \theta')} \qquad (26)$$

which may be obtained from equation (23) by eliminating $\frac{t}{w}$ in terms of l.

The relation between Φ', θ, and θ' is obtained by forming an equation for x similar to (24) for x', whence, with the value of h in (23), there results,

$$\tan.\ \Phi' = \frac{h}{x - x'} = \frac{\sec.\ \theta - \sec.\ \theta'}{\text{nap. log.}\left[\frac{\sec.\ \theta + \tan.\ \theta}{\sec.\ \theta' + \tan.\ \theta'}\right]} \qquad (27)$$

Table XII contains a series of values of Φ' deduced from equation (27) corresponding to such assumed values of θ and θ' as may occur in practice. With each value of Φ' is also tabulated the corresponding percentage of slack computed by means of equation (26). The results are rigorous representations of the properties of the catenary, and even though the wind effect has been omitted, the relations of the quantities concerned are such that the wind effect on the wire can not modify the percentage of slack, corresponding to given values of Φ' and θ', except by a quantity of secondary magnitude.

TABLE XII.—*Angular elevation and percentages of slack.*

		Θ' = Inclination of wire at reel.						
		0°.	10°.	20°.	30°.	40°.	50°.	60°.
$\Theta = 50°$	Slack, %	3.22	2.03	1.11	0.51	0.13	......	......
	Φ'	28.8°	32.9°	36.9°	41.0°	45.3°	......	......
$\Theta = 55°$	Slack, %	3.87	2.55	1.53	0.78	0.29	0.03	......
	Φ'	32.8°	36.6°	40.4°	44.2°	48.2°	52.6°	......
$\Theta = 60°$	Slack, %	4.53	3.10	1.97	1.11	0.50	0.13	......
	Φ'	37.2°	40.8°	44.3°	47.8°	51.4°	55.4°	......
$\Theta = 65°$	Slack, %	5.17	3.65	2.43	1.48	0.76	0.28	0.03
	Φ'	42.2°	45.4°	48.5°	51.7°	55.0°	58.5°	62.6°

TABLE XIII.—*Ratio of sag* $= S \div S'$.

	$S' = \Phi' - \Theta'$ = sag at reel.							
	2°	4°	6°	8°	10°	12°	14°	20°
$\Theta = 50°$	0.950	0.910	0.878	0.852	0.828	0.810	0.793	0.758
$\Theta = 55°$	0.942	0.894	0.856	0.836	0.800	0.779	0.760	0.718
$\Theta = 60°$	0.929	0.876	0.834	0.800	0.770	0.746	0.724	0.671
$\Theta = 65°$	0.918	0.854	0.804	0.766	0.731	0.705	0.681	0.627

The practical use made of Table XII is as follows: With Φ' and l we compute the approximate elevation of the kite from the equation, $h' = l \sin. \Phi'$; with Φ' and θ' we take from Table XII the corresponding percentage of slack; deducting from h' this same percentage of itself there results the actual elevation.

The ratios of the angles of sag, given in Table XIII, will be understood from what follows:

Angles of sag in partial ascensions.—In making efficiency tests we measure the angle of sag, S', at the reel, and desire to know the corresponding sag, S, at the kite. The ratio $S \div S'$ of these angles is nearly constant when S' is small, and it varies but little with different values of θ'. In computing these ratios we have used the relations $S' = \Phi' - \theta'$ and $S = \theta - \Phi'$, which are apparent from Fig. 77, and the values of Φ' deduced from equation (27).

Altitude as dependent upon pull.—Kites of different size pull with different forces. The maximum altitude a kite pulling with a given force t, at an inclination θ can attain is given by equation (10) thus,

$$h = t\,\frac{(1 - \cos.\theta)}{w} \qquad (10)$$

A kite that pulls twice as hard as another can, we see, attain twice the altitude. Moreover equation (7) shows that exactly twice the length of wire will be required. If instead of one large kite two smaller ones, each pulling half as hard but at the same angle, were made to pull, without interference, at the end of the line, it is plain that the combined action of the two kites would necessarily be equivalent to that of the large one in every respect. Suppose, however, the two kites were formed into a tandem in the usual fashion; we wish to know whether the top kite can then attain a greater, an equal, or a less elevation than that reached by the single equivalent kite.

Kites in tandem.—Some mention was made on page 121[1] of the greater steadiness of pull resulting from the use of two or more kites in tandem. This is an important matter in itself but does not directly concern us here as our analysis of the properties of the catenary proceeds upon the assumption that the tension on the wire is, in all cases, sufficiently steady to keep the resulting curve in a condition of complete static equilibrium. We assume further in our discussion of the distribution of kites in a tandem that all are subjected to the same wind force.

Two considerations arise in flying kites in tandem, namely, (1) having given a certain *pull*, acting in a certain direction, how shall this be employed to gain the maximum elevation? Shall the pull be concentrated and applied at the end of the kite line, or shall it be subdivided and distributed, and if so, how? (2) Having given a wire or line capable of sustaining a certain maximum safe-working tension, how shall it be employed with actual kites to attain the maximum elevation? We shall find that the same general equations will enable us to answer both these questions.

General equations for tandems.—Our equations will be sufficiently general if we assume that the different kites which go to make up the tandem are exactly equal in all respects, hence t and θ will represent the intensity and inclination of the pull of any of the kites.

Fig. 78 represents the forces acting at the point at which a second kite is attached to the line from the topmost or so-called pilot kite. Using a notation similar to that already employed, θ' and t' are, respectively, the inclination and pull of the portion of wire just above the point at which the second kite is attached. (θ' and t' result from the action of the pilot kite.) θ_2 and t_2 are respectively the inclination and pull of the portion of wire just below the point at which the second kite is attached; they represent the combined power of both kites. Constructing the parallelogram of forces between the tensions involved we obtain from trigonometrical relations,

$$t_2 = \sqrt{t^2 + t'^2 + 2\,t\,t'\cos.(\theta - \theta')} \qquad (28)$$

but,

$$t' = t\,\frac{\cos.\theta}{\cos.\theta'}$$

whence,

$$t_2 = t\sqrt{1 + \frac{\cos.^2\theta}{\cos.^2\theta'} + 2\,\frac{\cos.\theta}{\cos.\theta'}\cos.(\theta - \theta')} \qquad (29)$$

an equation which represents the resultant or combined pull of the two kites. The direction, θ_2, in which this pull is exerted, is obtained as follows: In the triangle of forces t, t', and t_2, let a be the angle included between the sides t' and t_2, then,

$$\sin.a = \frac{t}{t_2}\sin.(\theta - \theta')$$

From the diagram it is seen that,

$$\theta_2 = \theta' + a$$

In assuming that the second kite pulls at an angle θ and tension t at the point where it is attached to the main line we neglect, as we may without sensible error, the influence of the short connecting wire between the kite and main line.

The combined action of the two kites is, by the above equations, completely expressed in terms of the power of one kite. By a precisely similar process we may determine the effect of adding a third, a fourth, or any number of subordinate kites in tandem. As our object is to discover the best arrangement of kites in tandem it will suffice if we make comparisons on the basis of two kites only, since if there is a gain or a loss with two kites, a similar result will obtain with three or more.

Having attached a second kite to the line, let wire be unreeled until the portion next the reel becomes horizontal. It seems scarcely necessary to say that under no circumstances whatever should a second kite be attached that does not pull *above* the main line and thus tend to lift it. To attach a subordinate kite that pulls *below* the main line, and therefore drags it lower, would, obviously, be absurd if we aim to attain great elevations.

The total elevation attained by the tandem of two kites is, from equations (23) and (10),

$$H_2 = \frac{t}{w}\left(1 - \frac{\cos.\theta}{\cos.\theta'}\right) + \frac{t_2}{w}\left(1 - \cos.(\theta' + a)\right)$$

This equation can be transformed into the following:

$$H_2 = \frac{t}{w}\left\{\begin{array}{l} 1 - R + \sqrt{1 + R^2 + 2R\cos.(\theta - \theta')} \\ - \cos.\theta'\Big[R + \cos.(\theta - \theta')\Big] \\ + \sin.\theta'\sin.(\theta - \theta') \end{array}\right\} \qquad (30)$$

Where $R = \cos.\theta \div \cos.\theta'$.

Equation (30) expresses the maximum height that can be attained by two equal kites in terms depending upon the power of one of the kites and the point at which the second kite is attached to the main line.

The answers to questions (1) and (2), propounded above, are reached from a consideration of equations (28) and (30), as follows:

Best utilization of a given pull.—Assume that the two kites are attached side by side on the end of the main line. In this case,

$$\theta' = \theta, \text{ and } R = 1,$$

[1] MONTHLY WEATHER REVIEW for April, 1896.

whence the height becomes,

$$H_2 = \frac{2t}{w}(1 - \cos.\theta),$$

which means that, thus arranged, the two kites attain twice the elevation of one alone, as should be the case. To show the effects of attaching the second kite lower and lower down upon the main line, we will compute the relative heights attained when the second kite is attached after the line has sagged 10°, 20°, 30°, and including the case where the second kite is not attached until the top kite has carried up all the wire it can sustain, in which case $\theta' = 0$. We will assume that the kites pull at an angle $\theta = 55°$, and compute the elevations on the basis of the maximum height being 5,000 feet. The results are:

		Feet.	Loss. Feet.
Maximum effect, both kites at the top	....	$H_2 = 5{,}000$	......
Second kite attached where the sag is	10°, $\theta' = 45°$	$H_2 = 4{,}960$	40
" " " "	20°, $\theta' = 35°$	$H_2 = 4{,}850$	150
" " " "	30°, $\theta' = 25°$	$H_2 = 4{,}690$	310
" " " "	40°, $\theta' = 15°$	$H_2 = 4{,}470$	530
" " " "	50°, $\theta' = 5°$	$H_2 = 4{,}200$	800
" " " "	55°, $\theta' = 0°$	$H_2 = 4{,}040$	960

We find here that there is a continually increasing loss in the elevation attained when flying kites tandem, depending upon how much the line is permitted to sag before the second kite is attached. The best results correspond to the least sag of the wire between kites, and the maximum effect is obtained when $\theta' = \theta$; but this may mean either of two things: (1) that the kites are placed side by side at the end of the line or (2) that innumerable kites are attached along the line so close to each other that the line does not sag between them; in other words, that every particle of the line is acted upon by its kite just as it is by gravity. From the properties of the catenary thus brought out it results that the maximum service can not be obtained by flying kites in tandem. There are, however, from other considerations, many marked advantages in tandem flying, which consist in the greater steadiness of pull thereby secured under actual conditions of variable winds and greater security against accident; also the facility of using a large or small amount of sustaining surface as required by conditions of wind force. A special advantage results from the more equable distribution of the strain on the line, which otherwise, with a single kite, is a maximum at the top. In reeling in a long line of kites, it is an advantage to be able to lessen the opposing pull by the removal of one after another of the kites, rather than to have to wind them all in until the top end is reached. Notwithstanding such advantages, we must not lose sight of the marked superiority of one large kite at the end of the line when we aim to reach great elevations. Perhaps more will be gained by the use of two, to secure a more steady pull, than will be lost by virtue of the tandem arrangement, but these two kites are best placed near the top end of the line.

In connection with equation (30) it is instructive to notice the result when $\theta = 90°$. This is not attainable by kites but represents the case of captive balloons in perfectly still air, and upon the supposition that the balloons pull with a constant force at all elevations. No matter what value θ' may have between 0° and 90°, the equation shows that two balloons in tandem will go twice as high as one, etc. Furthermore, it will be found that equation (30) shows that less loss results in tandem arrangements the steeper the angle at which each kite pulls, that is, the greater the value of θ.

While equation (30) was deduced for but two kites it answers perfectly for the analysis of the effects of any number of kites, for having found the result of the combination of two kites this combination may be treated as one and combined with a third kite, etc.

Thus far our consideration of tandem flying has been confined wholly to the question, how much effect can be produced by a certain *pull*, and we have found that the maximum elevation is attained either by concentrating the pull wholly at the outer end of the line) and this is the only feasible arrangement) or by acting with a portion of the pull upon each particle of the wire just as gravity acts to pull it down.

Best utilization of a given line.—We will next consider the second question that arises in connection with tandems, namely, how to best employ a line of given strength to attain elevation. If we attach at the end of the given line a kite so large that its pull strains the line to its safe working limit, a second kite can not be attached without danger to the line, except at some point well down upon the line, where, by reason of the diminution of the tension in the line corresponding to its deeper and deeper sag, the combined pull of the two kites will not exceed the safe working strength of the line. The second kite can not, in any case, pull as much as the first kite, but may be larger and larger the more and more the line is permitted to sag. Equation (28), inverted, tells us how much a kite it is proposed to add, can pull without exceeding the strength of the line; t_2 in that equation becomes T, the working tension that the line can sustain; θ is the direction or inclination of the pull to the horizontal; θ' is the inclination and t' the tension of the wire at the point where the second kite is to be attached. The pull of the top kite has already been assumed to be $T =$ the strength of the line, and if θ'' is the inclination of this pull, then since

$$t' = T\frac{\cos.\theta''}{\cos.\theta'} = TR_1,$$

we get,

$$t = T\left[\sqrt{1 - R_1^2\left(1 - \cos.(\theta - \theta')\right)} - R_1\cos.(\theta - \theta')\right] \quad (31)$$

Equation (31) shows that the second kite can pull the hardest if it is attached where the main line has sagged down to the horizontal condition; that is, where $\theta' = 0$; but we have already found that this is the opposite of the conditions that must be satisfied to attain high elevations. The final conclusions are plain, namely: (1) To utilize a given pull to the best advantage it must be concentrated at the end of the line; (2), to attain the maximum elevation with a line of a given strength every part of it must be subjected to the maximum strain that it can sustain. In other words, we must attach the largest kite the line can carry at the top end, and then little by little, as the line sags and the tension thereon diminishes, the tension must be increased up to the safe limit by additional kites. Equation (31) applies broadly to all cases, and is independent of the weight of the line per unit length, which means that we need consider only T, the maximum safe working tension of the particular line that is employed, thus embracing the case where fine lines at the start are joined to stronger lines as the pull increases.

The wind-impressed catenary.—The special results brought out in the foregoing application of the properties of the catenary to kite flying are not strictly the exact results that will be attained in practice, because we have neglected to include the effect of the wind upon the wire, as we are forced to do by the limitation of our knowledge concerning its pressure upon long fine wires. It seems that some knowledge of this total effect might be gained by a comparison of the actual behavior of kites whose constants are fully known with those effects which our knowledge of the properties of the catenary show should result. The experimental work of the Weather Bureau has not as yet been carried sufficiently far to furnish data of this nature, but the matter has been carefully considered from this standpoint with a view of deducing what may be called a correction for wind effect on the wire.

The general nature of the action of the wind upon the wire, and its effects in modifying the catenary may be shown in a more or less satisfactory manner, as follows: Let Fig. 79 represent a catenary subjected to the action of the wind. Along the lower portions of the curve the wind effect is very slight, both because the inclination of the wire is small, and as a rule, the force of the wind near the ground is less than throughout the upper portions of the curve where the effect of the wind pressure upon the wire will be greater, both because of the steeper inclination of the latter and the greater force of the wind. We can not conceive that any appreciable friction arises in the flow of the wind over the wire, and as a result the wind pressure must be normal to the wire at every point. Let the pressure upon a small element of the wire at p be represented by the line $p\ v$. Also let $p\ g$ represent the weight of the same element. The effect will then be the same as if the element in question were acted upon by a single force $p\ r$, which is the resultant or combined effect of the two forces of wind and gravity. Drawing in a similar manner the resultant pressure at other points of the curve we see that the curve assumed by the wire must be one that results from the action of a nearly constant force, which tends to press the wire in a direction such as $P\ R$. If we consider only a portion of the catenary $A\ B$, such as might be involved in a partial ascension, we may plainly, with but little error, assume that the combined effects of wind and gravity act in the direction $P\ R$. In such a case the resulting curve will be sensibly the same as would result if we imagine that gravity alone acted, not in a vertical direction, but in the direction of the line $P\ R$. In other words, the general *form* of the curve will be given by the equations we have already deduced, if we imagine the origin of coordinates to be shifted to a new position as $O'\ Y'$, $O'\ X'$, which are parallel and perpendicular to the line $P\ R$. The *tension*, also, will be given approximately by those equations if we imagine w to be increased in proportion to the ratio of the lines pr to pg.

A very simple way of experimentally studying the effects that result from shifting the origin of coordinates in the manner mentioned as applied to kites, consists in laying off on a drawing board an inclined line, $A\ B$, representing the angular elevation of the kite under consideration. Draw $A\ B'$, forming the angle θ' with the horizontal, and representing the inclination of the wire at the reel. Placing the drawing board on edge and suspending a small chain next its surface we may produce in a beautiful manner the curve of the catenary that shall make the angle θ' at the reel, and we may locate its point of crossing the line at B. Fixing these points of the chain by pins or otherwise, it will be found that by raising one edge so that the board stands on its corner, thereby inclining the line $A\ B$ at different angles in a vertical plane we cause important changes in the inclination of the chain at its fixed points. In order to restore the original inclination, preserving still the same length of chain between the points $A\ B$, and the upper extremity of the chain upon the line $A\ B$, it will be found necessary to make the end B approach A as the line $A\ B$ is made more and more nearly horizontal. These suggestions suffice to show a very simple method that has been employed in several ways by the writer to study the wind affected catenary.

Until the experimental observations have given accurate data concerning the magnitude of the wind effect, it will not be desirable to attempt to deduce equations representing the combined action of wind and gravity. This interesting and important branch of the kite problem must be left for solution in the future.

In this discussion of the theory and practice of flying kites for scientific purposes, the writer has aimed to show how the well known forces of nature act in producing the more important effects commonly observed in kite flying and to point out those general and fundamental principles of physics and mechanics pertaining to kites, by the proper application of which principles we may expect to secure the maximum useful results according to the requirements of any particular case. The groundwork we have aimed to lay for this work is not as complete as we could wish, owing to the limited time available for the Weather Bureau kite experiments, but it is hoped to extend the work to more promising forms of kites than those that have thus far been employed.

The Editor of the REVIEW has shown a deep personal interest in both the kite experiments themselves and in the publication of this series of articles in the REVIEW and the writer wishes to acknowledge the benefits that have resulted from his careful revision of the manuscript and proof.

NOTES BY THE EDITOR.

THE ST. LOUIS TORNADO.

The great tornado of May 27, 1896, at St. Louis will long continue to furnish material for interesting articles and reminiscences, and the Editor hopes to select from these such items as may be of value to meteorology. The following is extracted from an excellent article in the Occident, by Prof. E. S. Holden, Director of the Lick Observatory. Professor Holden's remarks as to the forecasting of this tornado by the Weather Bureau are omitted, as these forecasts were disseminated much earlier and more widely than he was aware of.

During the month of May I was in St. Louis and was an eye witness of the destruction caused by the great tornado of May 27. In former years, 1881 to 1885, I was stationed at the Washburn Observatory of the University of Wisconsin (Madison), which lies in a region subject to tornadoes, and made it my business to study the causes and effects of these violent local storms so far as opportunity offered.

* * * * * * *

On the afternoon of May 27 I was in Forest Park in St. Louis with one of my daughters, about 3 o'clock, and the aspect of the sky at once reminded both of us of the "tornado-skies" we had been used to see. The upper sky was covered with a faint veil of grayish clouds parted into regular shapes roughly rectangular and some four or five degrees on a side. Between these figures were darker lanes, of gray-blue color. All around the visible horizon, from north, through west, to south, there was a rim of brassy lurid sky. In the west, or a little north of west and also in the southwest, were two heavy, black, towering clouds, roughly rectangular in figure. The aspect of these clouds was carefully watched to see if they sent out fibrous, twisted offshoots downward; and the brassy rim of sky next the horizon was examined to see if the color deepened toward green.

Either of these signs would, so far as our previous experience went, have indicated the coming of a veritable tornado. So long as they were absent the indications were for a severe thunderstorm later in the evening. It was "hurricane weather" and not "tornado weather" at first. A little before 4 o'clock the sky looked decidedly more threatening and I decided to take my daughter to the Southern Hotel, which I knew to be one of the stoutest structures in the city. My rooms were on the eastern side, the safer side, which relieved the slight feeling of anxiety somewhat.

* * * * * * *

My own experience was sufficiently exciting. As I have said, our rooms were on the lee side of the hotel facing a street running north and south. Loaded wagons in the street below were blown off their wheels, and the horses thrown down. The heavy iron cornice of a tall building in course of construction was hurled to the street and destroyed; another building was set on fire by lightning which entered by the wires on the roof; the hotel chimney-stack was blown down, causing a damage to glass, etc., of some $5,000 and wounding several employees, etc.

The wind first blew violently up the street (north) and after the center of the storm had passed it suddenly changed direction and blew south, and this change of direction made new wrecks. The winds in such a storm blow circularly round, or toward the vortex, and when

Chart V. Kite Experiments at the Weather Bureau.

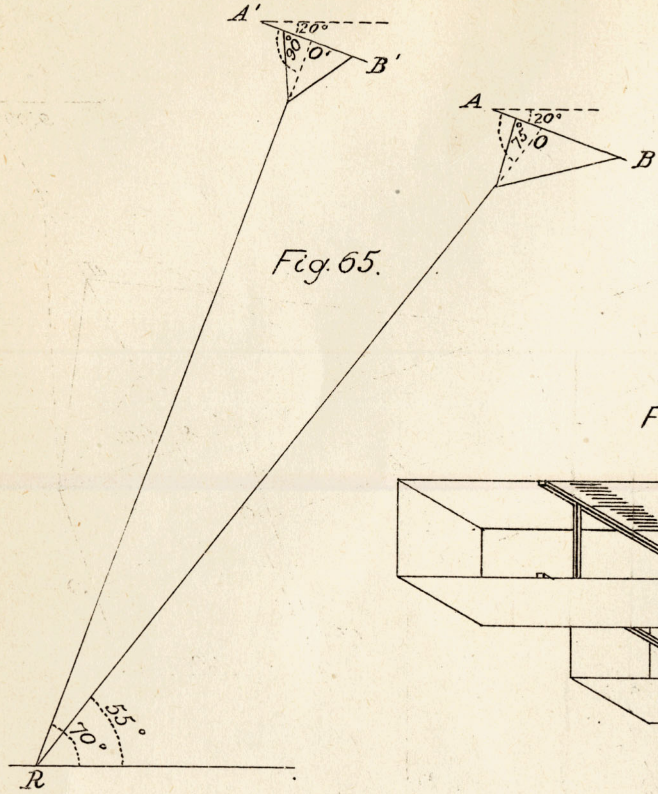
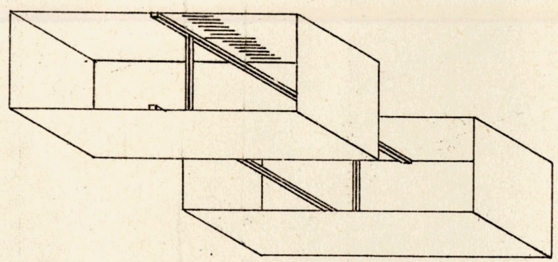


Fig. 65.

Fig. 66.


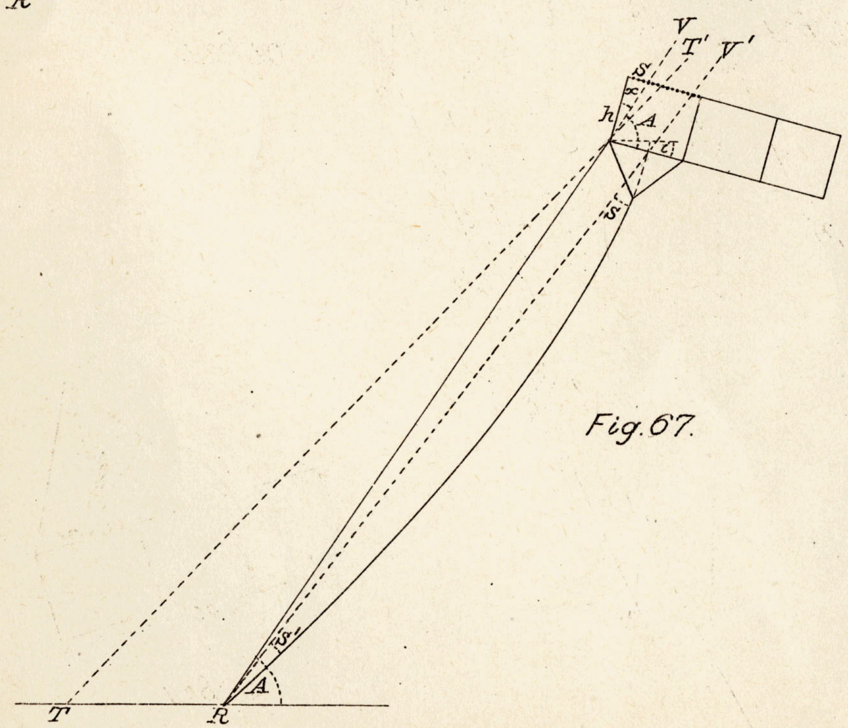


Fig. 67.

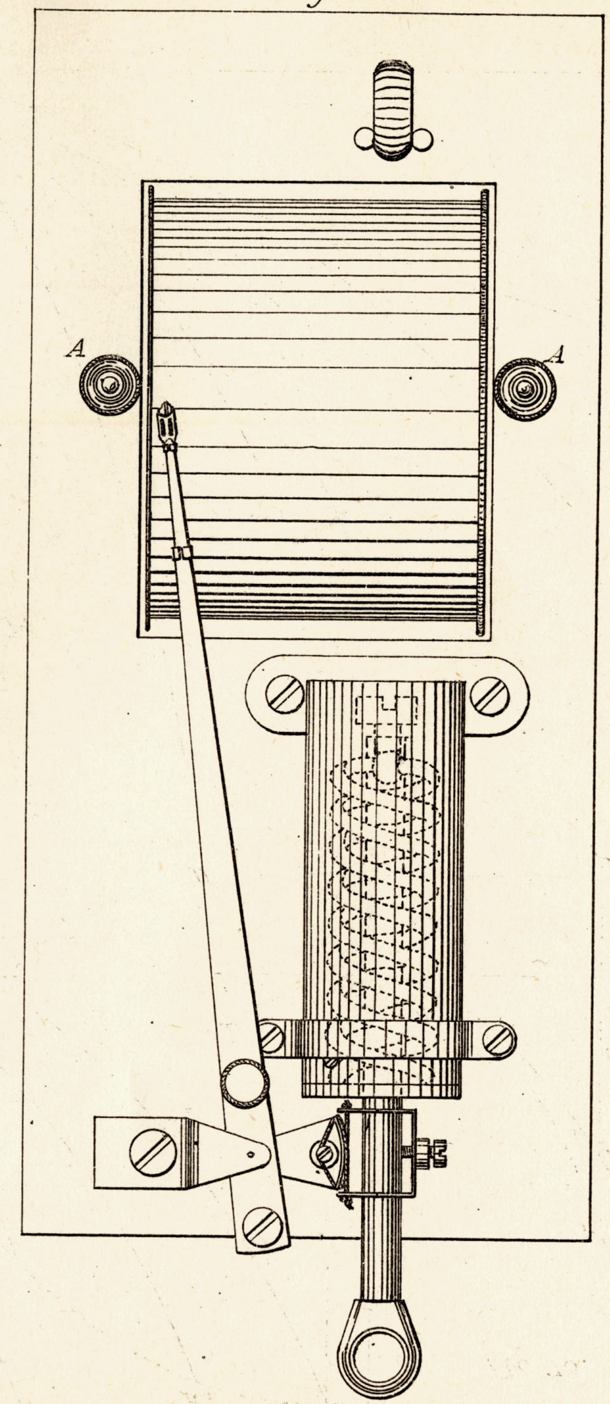
Fig. 68.


Chart VI. Kite Experiments at the Weather Bureau.

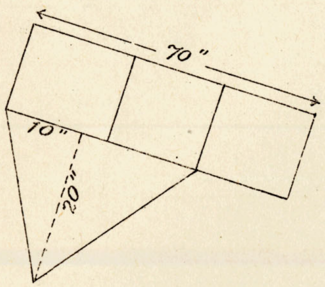


Fig. 69.

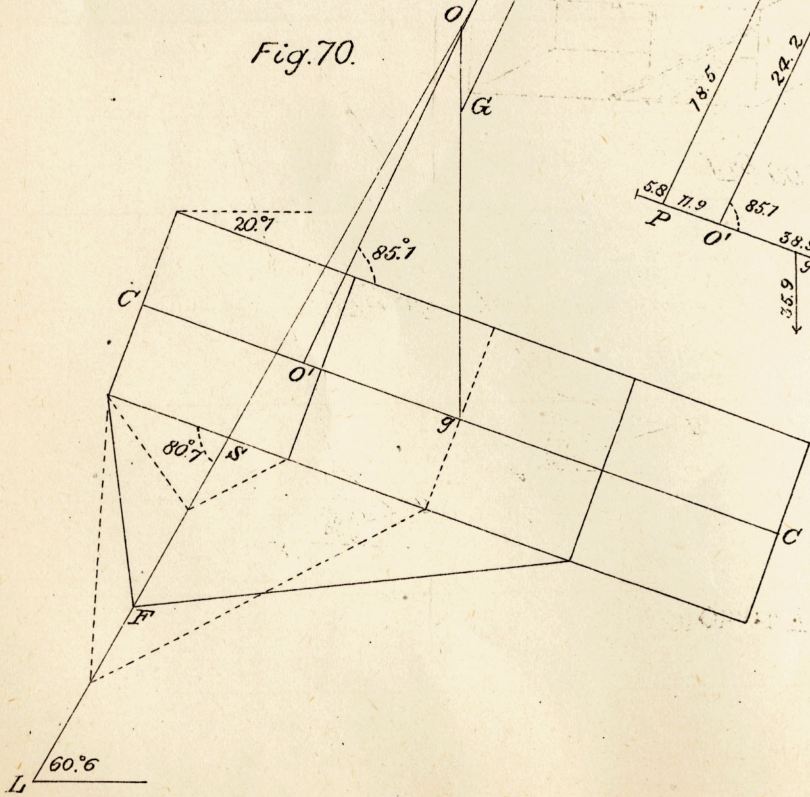


Fig. 70.

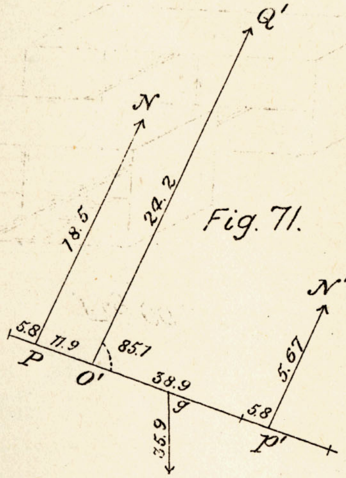


Fig. 71.

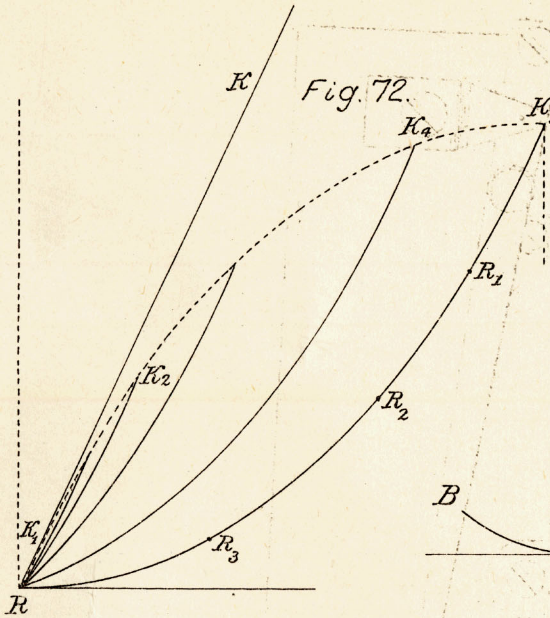


Fig. 72.

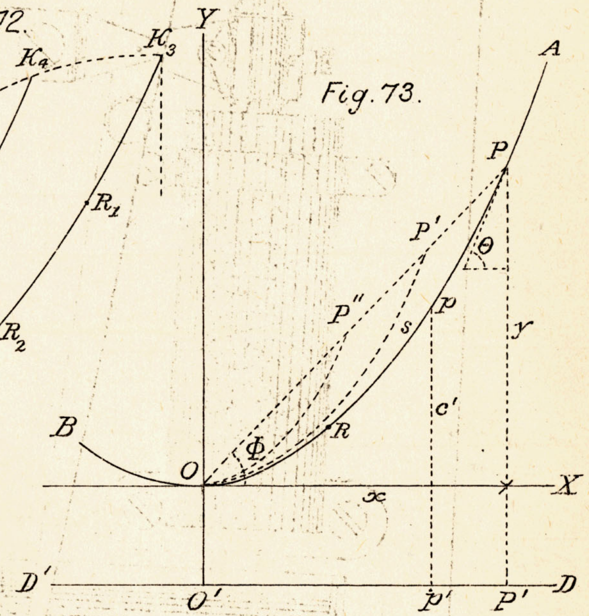


Fig. 73.

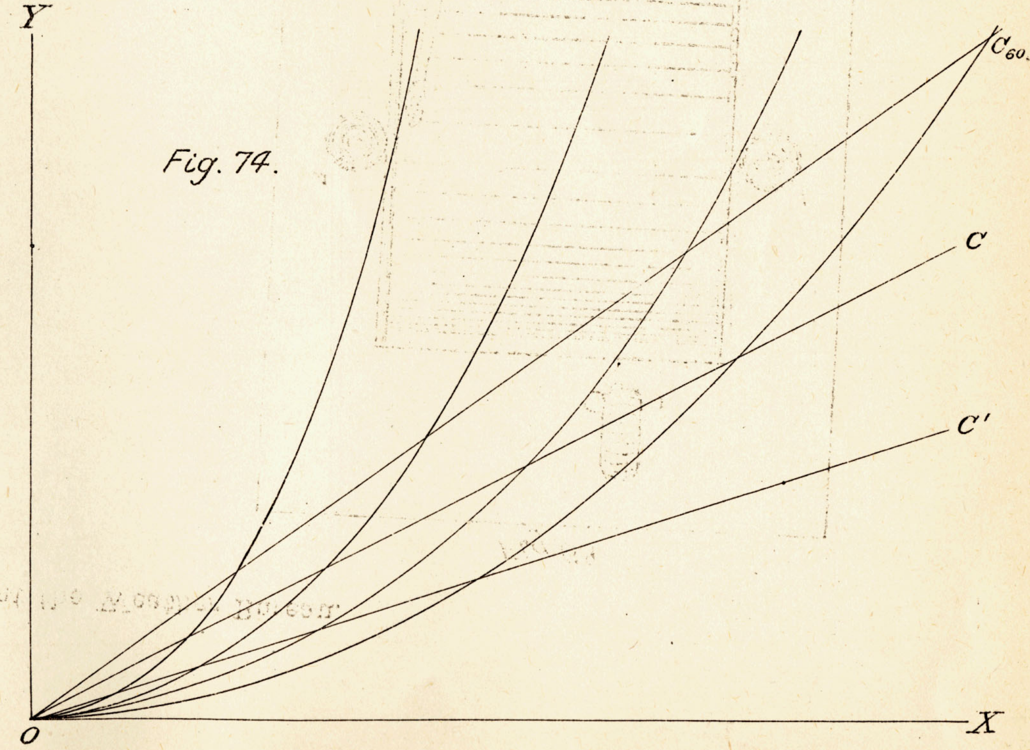


Fig. 74.

Chart VII. Kite Experiments at the Weather Bureau.

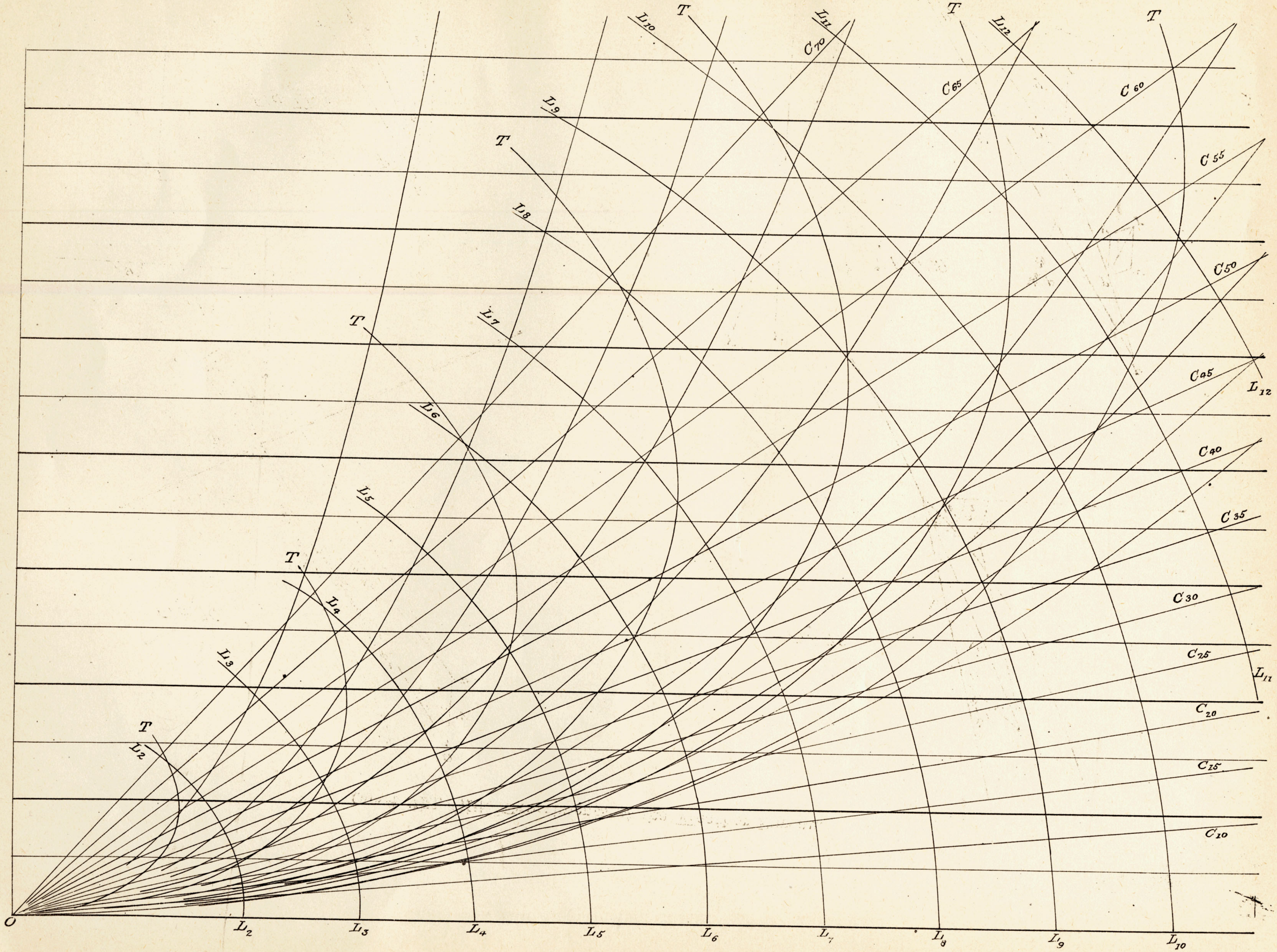


Chart VIII. Kite Experiments at the Weather Bureau.

Fig. 76.

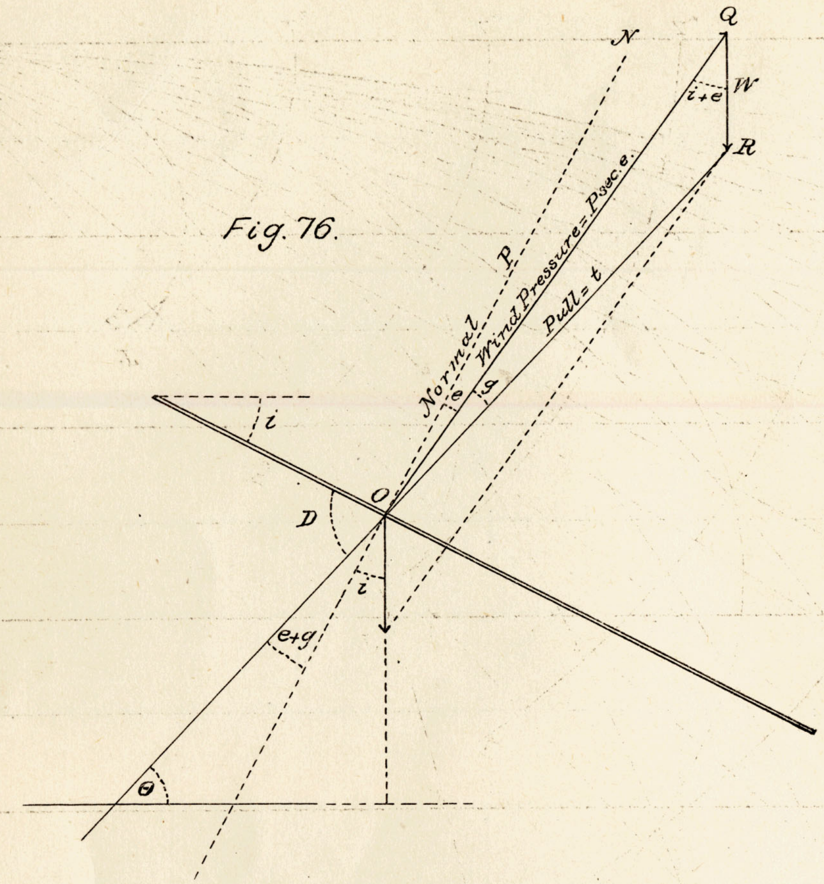


Fig. 78.

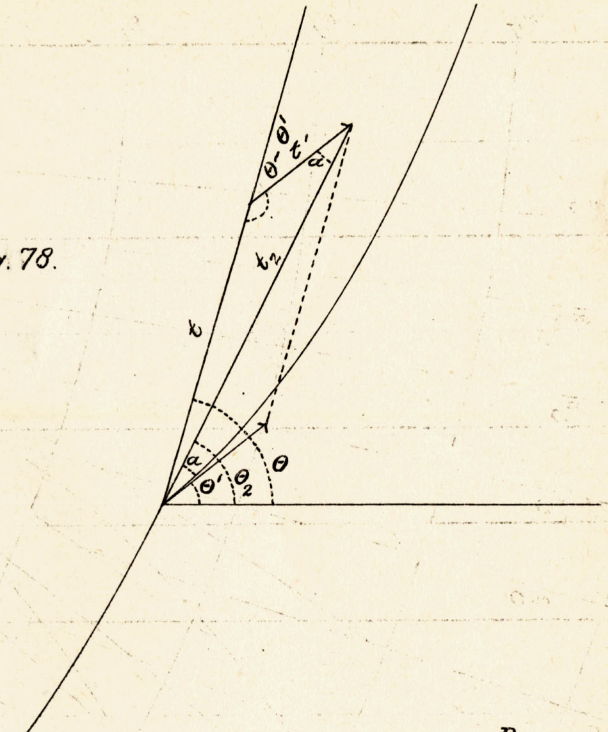


Fig. 77.

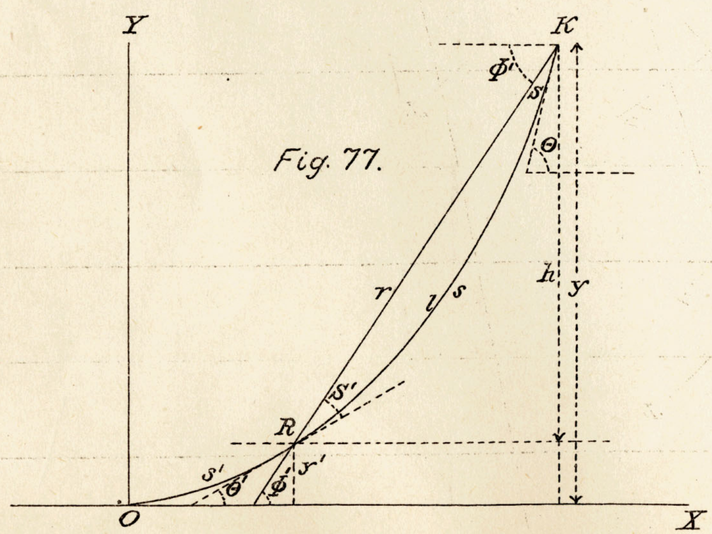


Fig. 79.